University of Birmingham

School of Physics and Astronomy

Y3/Y4 Formation and Evolution of Galaxies

Revision Problem Sheet 1

1. In a spherical galaxy, the circular velocity is given by

$$v_c^2 = \frac{a\, r^2}{(r^2+b^2)^{3/2}},$$

where a and b are constants.

   (a) Find an expression for the gravitational potential of this galaxy as a function of radius r. **[3]**

   (b) Show that for large radii $r \gg b$, the potential approaches that of a point mass. **[2]**

   (c) Using Poisson's equation, find the density of the galaxy as a function of the radius r. **[2]**

   (d) Using the substitution $r = b\tan\theta$, show that the mass of this galaxy is finite, and find its value. **[3]**

2. The relaxation time for the cluster may be expressed as

$$t_{\rm relax} \sim \frac{1}{8N\ln N}\frac{v^3 D^3}{(Gm)^2},$$

where D is the size of the cluster, N the number of stars, m their typical mass, v their typical speed, and G is the Gravitational constant.

   (a) Show that the ratio between relaxation time and crossing time is approximately $N/8\ln N$. [**4**]

   (b) A cluster of galaxies of radius 0.5 Mpc consists of 300 galaxies and has a one-dimensional velocity dispersion of 1000 km s^{-1}. Estimate both its crossing time and relaxation time. Compare these estimates to the age of the Universe and comment on the expected dynamical state of the cluster. [**6**]

3. Find the speed v_c of a star moving in a circular orbit about the centre of a singular isothermal sphere, of velocity dispersion σ (independent of radius), where the radial distribution of mass is given by $M(r) = 2\sigma^2 r/G$. **[10]**

4. A gas cloud of density ρ and sound speed c_s is collapsing under its own gravity. If the dispersion relation for acoustic waves is $\omega^2 = k^2 c_s^2 - 4\pi G\rho$, where the $k \equiv 2\pi/\lambda$ is the wavenumber, derive the Jeans length of the cloud in terms of the given quantities. What is the physical significance of the Jeans length in the context of the evolution of the cloud? **[10]**

5. Outline the observational evidence in favour of the presence of a supermassive black hole at the centre of the Milky way galaxy. Consider in particular the evidence from direct observations, the dynamics of the immediate surroundings of the Galactic centre, and high-energy observations resulting from the accretion properties of the candidate black hole. **[10]**