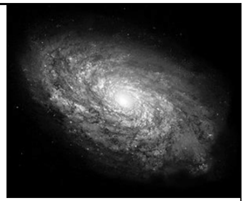
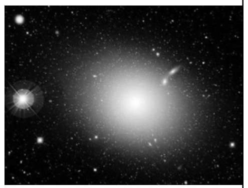
FORMATION AND EVOLUTION OF GALAXIES



Lecture 16

- **Galaxy Formation**
  - **Virial theorem**
  - Jeans instability for forming stars
  - Jeans length and mass



Somak Raychaudhury

The Virial theorem

$$\frac{1}{2}\cdot\frac{d^2 I}{dt^2} = 2T + V\,,$$

such that:

$$I = \sum_i m_i r_i^2\,,$$

where *I* is the spherical moment of inertia, and the r_i are distances from the COM. and

$$T = \frac{1}{2}\sum_i m_i v_i^2\,.$$

$$V = -\sum_{\text{pairs}} \frac{G m_i m_j}{r_{ij}}$$

Here, *T* is the total kinetic energy of the system.

Here, V is the total gravitational potential energy of the system.

Interpretation of the Virial Theorem

$$\frac{1}{2}\cdot\frac{d^2 I}{dt^2} = 2T + V\,.$$

- The term in *T* acts to 'support' the system against contraction or collapse.
- The term in V acts to contract the system, i.e., it is a 'collapse' term.

So:

1. If $2T + V > 0$ we have expansion.
2. If $2T + V < 0$ we have contraction.
3. If $2T + V = 0$ we have a stable system.

What if other forces are present, e.g., magnetic forces?

Then they will give rise to additional terms on the right-hand side.

The Virial Theorem

$$2\langle KE\rangle + \langle PE\rangle = 0$$

For a spherical cloud of temperature *T*,
- Assuming constant density
- ignoring magnetic fields
- neglecting rotation

Cloud's internal kinetic energy= $\langle KE\rangle = \frac{3}{2}NkT$

Cloud's total potential kinetic energy= $\langle PE\rangle = -\frac{3}{5}\frac{GM_c^2}{R_c}$

Condition for Collapse $2\langle KE\rangle < \langle PE\rangle$ or $3NkT < \frac{3}{5}\frac{GM_c^2}{R_c}$

Condition for collapse

$$3NkT < \frac{3}{5}\frac{GM_c^2}{R_c}$$

where N is the total number of particles. But N is just

$$N = \frac{M_c}{\mu m_H},$$

where μ is the mean molecular weight. Now, by the virial theorem, the condition for collapse ($2K < |U|$) becomes

$$\frac{3M_c kT}{\mu m_H} < \frac{3}{5}\frac{GM_c^2}{R_c}. \qquad (12.12)$$

The radius may be replaced by using the initial mass density of the cloud, ρ_0, assumed here to be constant throughout the cloud,

$$R_c = \left(\frac{3M_c}{4\pi\rho_0}\right)^{1/3}. \qquad (12.13)$$

Carroll & Ostlie 12.12

$$\frac{3M_c kT}{\mu m_H} < \frac{3}{5}\frac{GM_c^2}{R_c}. \qquad (12.12)$$

The radius of the cloud

$$R_c = \left(\frac{3M_c}{4\pi\rho_0}\right)^{1/3}. \qquad (12.13)$$

After substitution into Eq. (12.12), we may solve for the minimum mass necessary to initiate the spontaneous collapse of the cloud. This condition is known as the **Jeans criterion**:

$$M_c > M_J,$$

where

The Jeans mass

$$M_J \simeq \left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2} \qquad (12.14)$$

is called the **Jeans mass**. Using Eq. (12.13), the Jeans criterion may also be expressed in terms of the minimum radius necessary to collapse a cloud of density ρ_0:

$$R_c > R_J, \qquad (12.15)$$

where

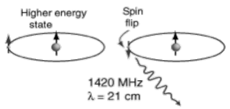
$$R_J \simeq \left(\frac{15kT}{4\pi G\mu m_H \rho_0}\right)^{1/2} \qquad (12.16)$$

is the **Jeans length**.

Interstellar Medium (ISM) - HI regions

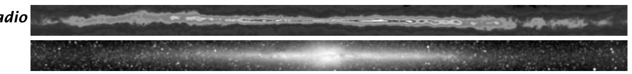
- Electrons have charge and spin, and so have a magnetic dipole moment.
- Pauli Exclusion Principle:. For a pair of electrons, if their spins are aligned, they have slightly more energy, than if they had opposite spin
- Energy difference is small ~6x10^{-6} *eV*, which is ~21 *cm* (radio waves).

From radio surveys of our galaxy we find lots of HI gas: about 1 solar mass of gas for every 10 solar masses of stars.

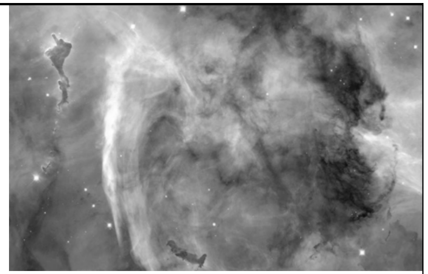


Radio

IR



Interstellar Medium (ISM) - HII regions



HST: Keyhole nebula

- HII regions are found near hot stars.
- UV photons from nearby stars ionize H atoms.
- When electrons and protons recombine, they generally emit Balmer lines.
- Also observe lines from other atoms (e.g., oxygen, sulphur, silicon, carbon, ...).
- HII regions are associated with active star formation.

Gravitational Collapse in the ISM

- The Jeans Mass

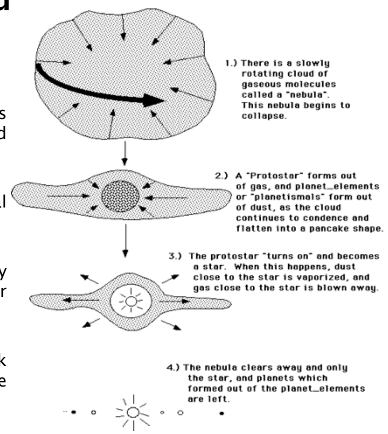
$$M_J = \left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2}$$

	Diffuse HI Cloud	H_2 Cloud Core
T	50 K	10 K
ρ	5×10^{8} m^{-3}	10^{10} m^{-3}
M_J	1500 $M_\odot$	10 $M_\odot$
M_c	1-100 $M_\odot$	10-1000 $M_\odot$

- We know from the ***Jeans Criterion*** that if $M_c > M_J$ collapse occurs.
- Substituting the values from the table into gives:
  - Diffuse HI cloud: $M_J \sim 1500\ M_{Sun}$ => stable as $M_c < M_J$.
  - Molecular cloud core: $M_J \sim 10\ M_{sun}$ => unstable as $M_c > M_J$.
- So deep inside molecular clouds the cores are collapsing to form stars.

Cloud Collapse and Star/Planet Formation

- Jeans cloud collapse equations describe the conditions required for an ISM cloud to collapse.
- As a cloud collapses, central temperature increases.
- This is accompanied by spinning-up of the central star (to conserve AM).
- If the cloud is rotating, the disk also flattens into an oblate spheroid.



Time-scale for collapse

- The collapse time-scale t_{ff} when $M > M_J$ is given by the time a mass element at the cloud surface needs to reach the centre.
- In free-fall, an mass element is subject to acceleration $g = \frac{GM}{R^2}$
- By approximating R using $R^3 \sim M/\rho \Rightarrow t_{ff} \approx (G\rho)^{-1/2}$
- Higher density at cloud centre = > faster collapse.
- For typical molecular cloud, $t_{ff} \sim 10^3$ years (ie very short).

Problem of star formation efficiency

Gas in the galaxy should be wildly gravitationally unstable. It should convert all its mass into stars on a free-fall time scale:

$$t_{\rm ff} = \sqrt{\frac{3\pi}{32G\rho}} = \frac{3.4\times 10^{10}}{\sqrt{n}}\ {\rm yr}$$

For interstellar medium (ISM) in our galaxy: $n \approx 2\times 10^{5}\ {\rm m^{-3}}$

$$t_{\rm ff} = 8\times 10^{6}\ {\rm yr}$$

Total amount of molecular gas in the Galaxy: $\sim 2\times 10^{9}\, M_{\rm sun}$

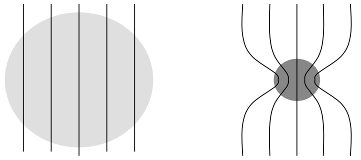
Expected star formation rate: $\sim 250\ M_{\rm sun}/{\rm year}$

Observed star formation rate: $\sim 3\ M_{\rm sun}/{\rm year}$

Something slows star formation down...

Magnetic field support

In presence of B-field, the stability analysis changes.
Magnetic fields can provide support against gravity.



Replace Jeans mass with critical mass, defined as:

$$M_{\rm cr} = 0.12 \frac{\Phi_{\rm M}}{G^{1/2}} \approx 10^3 M_{\rm sun} \left(\frac{|\mathbf{B}|}{3\,\rm nT} \right) \left(\frac{R}{2\,\rm pc} \right)^2$$

nT is nano-Tesla!

Magnetic field support

Consider an initially stable cloud. We now compress it. The density thereby increases, but the mass of the cloud stays constant.

Jeans mass *decreases:*

$$M_J \propto \frac{1}{\sqrt{\rho}}$$

If no magnetic fields: there will come a time when $M > M_J$ and the cloud will collapse.

But M_{cr} stays constant (the magnetic flux will be frozen in the cloud)

So if B-field is strong enough to support a cloud, no compression will cause it to collapse.