

FORMATION AND EVOLUTION OF GALAXIES

Lecture 20

- Galaxy formation in an expanding Universe
 - Jeans mass in a matter dominated Universe
 - Jeans mass in a radiation dominated Universe

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When did galaxies form: a rough estimate (1)

Consider a small ($\delta\rho/\rho \ll 1$) spherical perturbation of radius r :

$$M(r) \equiv (4\pi/3)r^3\bar{\rho}_{\text{matter}} = (4\pi/3)r^3(3H^2/8\pi G)\Omega_{\text{matter}}$$

For $H_0 = 100h$ km/s/Mpc,

$$M(r)/M_{\text{sun}} \approx 1.16 \times 10^{12} h^2 \Omega_{\text{matter}} r_{\text{Mpc}}^3$$

For $h=0.7$ and $\Omega_{\text{matter}}=0.25$,

$$M(r)/M_{\text{sun}} \approx 1.4 \times 10^{11} r_{\text{Mpc}}^3$$

i.e. an $\sim L^$ galaxy coalesced from matter within a comoving volume of roughly $\sim 1\text{-}1.5$ Mpc radius*

When did galaxies form: a rough estimate (2)

The typical radius of the visible parts of an L^* galaxy at $z=0$ is 10-20 kpc

However, we know that is embedded in a DM halo 5-10 times larger, say 75 kpc

→ The density enhancement represented by such a galaxy is thus

$$\delta = \delta\rho/\rho_{\text{matter}} \sim (1500/75)^3 \sim 8000$$

Since ρ_{matter} evolves like $(1+z)^3$, the density fluctuation was of order $\delta \sim 1$ ("epoch of formation") when $(1+z)^3 \sim 8000$ (*), i.e.

→ an L^* galaxy could not have separated from Hubble flow before $z \sim 20$

(*) from the analysis of the spherical infall model, such a perturbation would "turn around" at a $z \sim 10$

Jeans mass in an expanding Universe

- The Jeans length depends on the sound speed and density of matter

$$2r \approx \sqrt{\frac{15}{\pi}} \sqrt{\frac{c_s^2}{G\rho}} \approx \lambda_J, \text{ where } \lambda_J \equiv c_s \sqrt{\frac{\pi}{G\rho}}.$$

Early on, while the Universe is radiation-dominated, the density $\rho_r = a_B T^4/c^2$ is low and the pressure is high, with $c_s = c/\sqrt{3}$. So Equation 8.71 gives

$$\lambda_J = c^2 \left(\frac{\pi}{3Ga_B T^4} \right)^{1/2} \propto T^{-2}. \quad (8.72)$$

The *Jeans mass* \mathcal{M}_J is the amount of matter in a sphere of diameter λ_J :

$$\mathcal{M}_J \equiv \frac{\pi}{6} \lambda_J^3 \rho_m, \quad (8.73)$$

Jeans mass in an expanding Universe

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- The energy density of an ordinary sound wave is positive. However, the gravitational energy density of a sound wave is negative, since the enhanced attraction in the compressed regions overwhelms the reduced attraction in the dilated regions.
- The Jeans instability sets in where the net energy density becomes negative, so that the system can evolve to a lower energy state by allowing the wave to grow, and thus the system to fragment.

Jeans mass: radiation dominated

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Consider, for instance, the “radiation” component of the universe. With $w = 1/3$, the sound speed in a gas of photons or other relativistic particles is

$$c_s = c/\sqrt{3} \approx 0.58c. \quad (12.25)$$

The Jeans length for radiation in an expanding universe is then

$$\lambda_J = \frac{2\pi\sqrt{2}}{3} \frac{c}{H} \approx 3.0 \frac{c}{H}.$$

Jeans mass: matter dominated

(ii) After recombination, radiation pressure is negligible, so the sound speed

$$c_s^2 = dP/d\rho = \left(\frac{k_B T}{\mu m_H} \right)^{1/2},$$

where μ is the relative mass of each particle. Thus, the Jeans Mass

$$M_J = \frac{\pi}{6} \rho_m \left(\frac{\pi k_B T}{G \rho_m m_H} \right)^{3/2}.$$

The Jeans mass in the radiation dominated era is of the order of $10^{16} M_\odot$ (scale of galaxy clusters). Adiabatic baryonic perturbations with scale sizes smaller than superclusters cannot grow before recombination.

After recombination, the Jeans mass abruptly falls to globular cluster masses ($\sim 10^5 M_\odot$). At this point, all scales of mass $> 10^5 M_\odot$ become unstable, and perturbations

Jeans mass before decoupling

if we regard the baryons as a minor contaminant, the Jeans length of the photon-baryon fluid was roughly the same as the Jeans length of a pure photon gas:

$$\lambda_J(\text{before}) \approx 3c/H(z_{\text{dec}}) \approx 0.6 \text{ Mpc} \approx 1.9 \times 10^{22} \text{ m}. \quad (12.27)$$

The *baryonic Jeans mass*, M_J , is defined as the mass of baryons contained within a sphere of radius λ_J :

$$M_J \equiv \rho_{\text{bary}} \left(\frac{4\pi}{3} \lambda_J^3 \right). \quad (12.28)$$

Immediately before decoupling, the baryonic Jeans mass was

$$\begin{aligned} M_J(\text{before}) &\approx 5.0 \times 10^{-19} \text{ kg m}^{-3} \left(\frac{4\pi}{3} \right) (1.9 \times 10^{22} \text{ m})^3 \\ &\approx 1.3 \times 10^{49} \text{ kg} \approx 7 \times 10^{18} M_\odot. \end{aligned} \quad (12.29)$$

This is approximately 3×10^4 times greater than the estimated baryonic mass of the Coma cluster, and represents a mass greater than the baryonic mass of even the largest supercluster seen today.

Jeans mass after decoupling

Now consider what happens to the baryonic Jeans mass immediately after decoupling. Once the photons are decoupled, the photons and baryons form two separate gases, instead of a single photon-baryon fluid. The sound speed in the photon gas is

$$c_s(\text{photon}) = c/\sqrt{3} \approx 0.58c . \quad (12.30)$$

The sound speed in the baryonic gas, by contrast, is

$$c_s(\text{baryon}) = \left(\frac{kT}{mc^2} \right)^{1/2} c . \quad (12.31)$$

At the time of decoupling, the thermal energy per particle was $kT_{\text{dec}} \approx 0.26 \text{ eV}$, and the mean rest energy of the atoms in the baryonic gas was $mc^2 = 1.22m_p c^2 \approx 1140 \text{ MeV}$, taking into account the helium mass fraction of $Y_p = 0.24$. Thus, the sound speed of the baryonic gas immediately after decoupling was

$$c_s(\text{baryon}) \approx \left(\frac{0.26 \text{ eV}}{1140 \times 10^6 \text{ eV}} \right)^{1/2} c \approx 1.5 \times 10^{-5} c , \quad (12.32)$$

only 5 kilometers per second. Thus, once the baryons were decoupled from the photons, their associated Jeans length decreased by a factor

$$F = \frac{c_s(\text{baryon})}{c_s(\text{photon})} \approx \frac{1.5 \times 10^{-5}}{0.58} \approx 2.6 \times 10^{-5} . \quad (12.33)$$

Decoupling causes the baryonic Jeans mass to decrease by a factor $F^3 \approx 1.8 \times 10^{-14}$, plummeting from $M_J(\text{before}) \approx 7 \times 10^{18} M_\odot$ to

$$M_J(\text{after}) = F^3 M_J(\text{before}) \approx 1 \times 10^5 M_\odot . \quad (12.34)$$

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This is comparable to the baryonic mass of the smallest dwarf galaxies known, and is very much smaller than the baryonic mass of our own Galaxy, which is $\sim 10^{11} M_\odot$.

The abrupt decrease of the baryonic Jeans mass at the time of decoupling marks an important epoch in the history of structure formation. Perturbations in the baryon density, from supercluster scales down to the size of the

smallest dwarf galaxies, couldn't grow in amplitude until the time of photon decoupling, when the universe had reached the ripe old age of $t_{\text{dec}} \approx 0.35 \text{ Myr}$. After decoupling, the growth of density perturbations in the baryonic component was off and running. The baryonic Jeans mass, already small by cosmological standards at the time of decoupling, dropped still further with time as the universe expanded and the baryonic component cooled.

REVISION

- Timescales
- Using proper units in numerical calculations
- Potential and density pairs
- Virial theorem
- Spiral vs elliptical galaxies
- Star formation- Jeans length, measuring SFR
- How to stop star formation. Late to early type transformation
- Galaxy formation- Jeans length
- Black holes and AGN