Formation and Evolution of Galaxies

Spring 2012

REVISION

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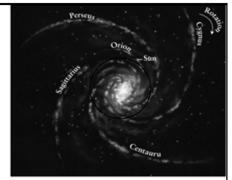
- Course resources
  - Website
  - Books:
    - Sparke and Gallagher, 2nd Edition
    - Carroll and Ostlie, 2nd Edition

An easy way for numerical calculations



- How long does it take for the Sun to go around the galaxy?
- The Sun is travelling at v=220 km/s in a mostly circular orbit, of radius r=8 kpc

Use another system of Units:

- ◆ Assume G=1
- ◆ Unit of distance = 1kpc
- ◆ Unit of velocity= 1 km/s
- ◆ Then Unit of time becomes 10^9 yr
- ◆ And Unit of Mass becomes $2.3 \times 10^5\ M_\odot$

So the time taken is $2\pi r/v = 2\pi \times 8/220$ time units

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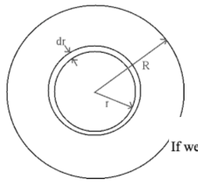
12. The Sun's circular velocity around the centre of the Milky Way galaxy is measured to be $v_c = 220$ km s^{-1} at its radius of $R = 8$ kpc. Assuming that the distribution of matter in the Galaxy is spherically symmetric, what is the mass of the Galaxy interior to this radius (in solar masses)? How many revolutions around the centre of the Galaxy will the Sun have made in the 4.5 Gyr of its life? **[10]**

R =8 kpc and $v_t = 220$ km/s, we have $v_t^2/r = GM(<r)/r^2$. So the mass interior to the orbit is $M(<r) = v_t^2 r/G$. In the units taught in class, G=1, and unit of length, speed, time and mass are 1 kpc, 1 km/s, 10^9yr, and $2.3 \times 10^5 M_\odot$ respectively. So $M(<r) = 220^2 \times 8 = 8.9 \times 10^{10} M_\odot$. Also time for each orbit=$2\pi \times 8/220 = 2.2 \times 10^8$ yr. So the Sun has gone around the Galaxy 20 times. [7+3]

Gravitational potential energy



If we now integrate over all radii, we get the gravitational potential energy of the whole sphere:

$$U = -\frac{16G\pi^2\bar{\rho}^2}{3}\int_0^R r^4 dr$$

$$= -\frac{16G\pi^2\bar{\rho}^2}{3}\frac{R^5}{5}$$

$$\bar{\rho} = \frac{M}{\frac{4}{3}\pi R^3}$$

gravitational potential energy of a uniform sphere

$$U = -\frac{3}{5}\frac{GM^2}{R}$$

Measuring the mass of a galaxy cluster

The Virial theorem $2\langle T\rangle + \langle V\rangle = 0$

KINETIC ENERGY (T): Observers measure radial velocities v_r of galaxies from Doppler shifts in their spectra. The mean of all redshifts in a cluster $\langle v\rangle$ would yield the mean radial velocity of the cluster with respect to the observer (largely due to the Hubble expansion of the Universe). The dispersion σ_r^2 of the measured values of v_r about this mean would be a measure of the K.E. of the galaxies, so

$$T = 3 \times \frac{1}{2} M \sigma_r^2,$$

where M is the combined mass of all the galaxies, the factor 3 accounting for the fact that one measures only the radial component of the velocities of the galaxies, whereas the kinetic energy would depend on the net spatial velocity v of each galaxy, and statistically $\langle v^2\rangle = 3\langle \sigma_r^2\rangle$.

Use the virial theorem to find a relation between the mass of cluster of galaxies, its radius R and σ_r, assuming it to be a unform sphere. Consider the Coma cluster to be a spherical cluster of galaxies of radius $R = 2$ Mpc, and one-dimensional velocity dispersion $\sigma = 1200$ km/s, find the mass of the cluster in solar masses.

Virial theorem: $2T + V = 0$. For a spherical cluster, $V = -\frac{3}{5}\frac{GM^2}{R}$ and $T = \frac{1}{2}M\sigma^2$. But since measured velocity dispersion is only radial, this $\sigma^2 = 3\sigma_r^2$. So $M(R) = 5\sigma_r^2 R/G$.

Remember our units: distance 1 kpc, speed 1 km/s, time 10^9 *yr,* $G = 1$, *and mass* $2.3 \times 10^5, M_\odot$.

Plugging $\sigma_r = 1273$ km/s, and $R = 2$ Mpc into the result in (b), $M = 3.5 \times 10^{15} M_\odot$.

Virial theorem

The luminosity of an elliptical roughly scales as its average velocity dispersion as the *Faber-Jackson relation*

$$L \propto \sigma^4, \qquad (6.2)$$

and is often used to measure distances to ellipticals (this was the relation used by the 'Seven Samurai', for instance, in the study that found evidence for a 'Great attractor' in our neighbourhood). But it turns out that all ellipticals don't obey the F-J relation in the same way–the surface brightness of the elliptical plays a role as well.

If we assume that the velocity dispersion of stars σ and the M/L ratio is constant throughout an elliptical galaxy, we can use the virial theorem to infer a relation between the global measurable parameters of ellipticals. From the virial theorem we have $2T + V = 0$, or, approximately,

$$Mv^2 - \frac{3}{5}\frac{GM^2}{R} = 0.$$

This gives mass $M \sim v^2 R/G$. The *mass surface density* then should go as

$$\Sigma \equiv M/R^2 \sim \frac{v^2}{GR},$$

Newton's shell theorems

THEOREM [**Newton I**] *The net gravitational force exerted by a spherical shell of matter on a particle at a point inside the shell is identically zero.* □

THEOREM [**Newton II**] *The gravitational force on a particle that lies outside a closed spherical shell of matter is the same as it would be if all the shell's matter were concentrated into a point at the centre of the shell.* □

to calculate the gravitational potential due to an arbitrary spherically symmetric mass distribution of density $\rho(\mathbf{x})$, which we can split into two parts: the contribution from shells with $\mathbf{r} < \mathbf{x}$ and with $\mathbf{r} > \mathbf{x}$:

$$\Phi(\mathbf{x}) = -4\pi G\left[\frac{1}{x}\int_0^x \rho(r)\, r^2\, dr + \int_x^\infty \rho(r)\, r\, dr\right]. \qquad (2.5)$$

Potential-density pairs

2. In a spherical galaxy, the circular velocity is given by

$$v_c^2 = \frac{a\,r^2}{(r^2+b^2)^{3/2}},$$

where a and b are constants.

(a) Find an expression for the gravitational potential of this galaxy as a function of radius r.

$$\frac{d\Phi}{dr} = -\frac{v_c^2}{r} = -\frac{a\,r}{(r^2+b^2)^{3/2}}.$$

Integrating from $r=0$ to r,

$$\Phi(r) = -\frac{a}{(r^2+b^2)^{1/2}}.$$

(b) Show that for large radii $r \gg b$, the potential approaches that of a point mass.

(c) Using Poisson's equation, find the density of the galaxy as a function of the radius r.

Potential-density pairs

(c) Using Poisson's equation, find the density of the galaxy as a function of the radius r.

Use Poisson's Equation in spherical coordinates. Since the problem is spherically symmetric, only the d/dr terms are relevant.

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = 4\pi G\rho$$

Use the expression of v_c^2 given, noticing that $r(d\Phi/dr) = v_c^2$,

$$r^2\frac{d\Phi}{dr} = \frac{a\,r^3}{(r^2+b^2)^{3/2}}.$$

$$\rho(r) = \frac{a}{4\pi G}\frac{3b^2}{(r^2+b^2)^{5/2}},$$

(d) Using the substitution $r = b\tan\theta$, show that the mass of this galaxy is finite, and find its value.

$$\text{Total Mass} = \int_0^\infty 4\pi r^2\rho dr = \frac{3ab^2}{4\pi G}.4\pi\int_0^\infty \frac{r^2dr}{(r^2+b^2)^{5/2}}.$$

With $r = b\tan\theta$, one gets $r^2+b^2 = b^2\sec^\theta$, so the integral turns into

$$\frac{3a}{G}\int_0^{\pi/2}\sin^2\theta\cos\theta d\theta = \frac{a}{G}.$$

Effective potential

$$mr\dot{\theta}^2 = mv_\theta^2/r,$$

the familiar centrifugal force. The corresponding potential is given by

$$V'(r) = V(r) + \tfrac{1}{2}\frac{L^2}{mr^2}. \qquad (3.6)$$

We will call this the *effective potential*. Furthermore, the energy conservation relation implies that the total energy

$$E \equiv V'(r) + \tfrac{1}{2}m\dot{r}^2 = V(r) + \tfrac{1}{2}\frac{L^2}{mr^2} + \tfrac{1}{2}m\dot{r}^2 \qquad (3.7)$$

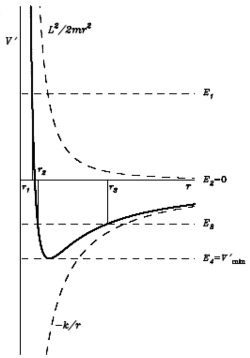
THE INVERSE-SQUARE CENTRAL FORCE

For a inverse-square central force, *e.g.* gravitation,

$$F(r) = -\frac{k}{r^2}, \qquad V(r) = -\frac{k}{r},$$

and the corresponding effective potential is

$$V'(r) = -\frac{k}{r} + \frac{L^2}{2mr^2}.$$



Two special cases:

1) $\Phi = \frac{1}{2}\Omega^2 r^2$ (simple harmonic oscillator, the potential of a uniform density sphere). The orbits are closed ellipses centered on the origin, $\Delta\psi = \pi$ in one radial period.

2) The Keplerian potential $\Phi = -GM/r$. Orbits are closed ellipses with the focus at the origin. The ellipse is given by

$$r = a(1-e^2)/[1 + e\cos(\psi - \psi_\circ)]$$

where a and e are related to the energy E and angular momentum L by

$a = L^2/GM(1-e^2), \quad E = -GM/2a$

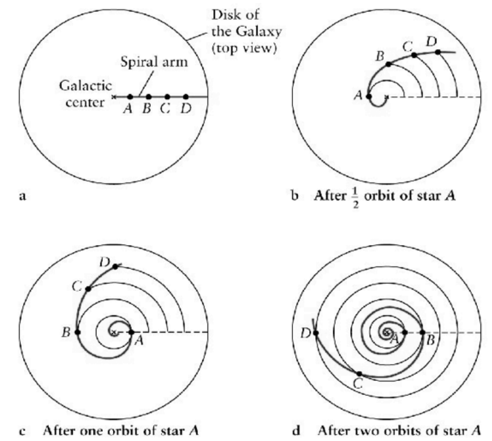
$r_{max}, r_{min} = a(1 \pm e)$ and $T_r = T_\psi = 2\pi\sqrt{a^3/GM} = T_r(E)$

$\Delta\psi = 2\pi$ in one radial period.

Bertrand's theorem

Spiral arms are caused by density waves that sweep around the Galaxy.

The Winding Paradox (dilemma) is that if galaxies rotated like this, the spiral structure would be quickly erased.



To establish the existence of SMBHs

- Stellar kinematics in the core of the galaxy
- Optical spectra: the width of the spectral line from broad emission lines
- X-ray spectra: The iron Kα line is seen is clearly seen in some AGN spectra
- The bolometric luminosities of the central regions of some galaxies is much larger than the Eddington luminosity
- Variability in X-rays: Causality demands that the scale of variability corresponds to an upper limit to the light-travel time

Schwarzschild radius

So the radial distance between two nearby points on the same radial line ($d\theta=0$, $d\varphi=0$) measured simultaneously ($dt=0$) is the proper distance

$$d\mathcal{L} = \sqrt{-(ds)^2} = \frac{dr}{\sqrt{1 - 2GM/rc^2}}.$$

The proper time $d\tau$ recorded by clock at radial distance r is related to the time t recorded at infinity

$$d\tau = \frac{ds}{c} = dt\sqrt{1 - \frac{2GM}{rc^2}},$$

When the radial coordinate of the star's surface has collapsed to

$$R_S = 2GM/c^2, \quad (17.27)$$

called the **Schwarzschild radius**, the square roots in the metric go to zero. The resulting behavior of space and time at $r = R_S$ is remarkable. For example, according to Eq. (17.17), the proper time measured by a clock at the Schwarzschild radius is $d\tau = 0$. Time has slowed to a complete stop, as measured from a vantage point that is at rest a great distance away.[14] From this viewpoint, *nothing ever happens at the Schwarzschild radius*!

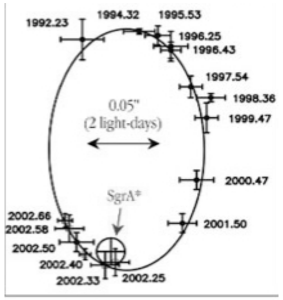
Schwarzschild radius

$$R_S = \frac{2GM}{c^2}$$

Size of a black hole

Mass	$R_S = 3\left(\frac{M}{M_\oplus}\right)$ km	Innermost stable orbit=$3R_S$
1 Earth mass	2 cm	6 cm
1 Solar mass	3 km	9 km
10 Solar masses	30 km	90 km
1 million Solar masses	3 million km ~ 4 x	9 million km Solar radius!

S2: star with orbital period 15.2 yr



- For S2, Eccentricity =0.87
- Perigalacticon distance = 120 AU
  r_p = 120 x 1.5 x 10^8 km

$$a = \frac{r_p}{1-e} = 1.4 \times 10^{14}\ \mathrm{m}$$

- From Kepler 3,

$$\frac{4\pi^2 a^3}{GP^2} \simeq 7 \times 10^{36}\ \mathrm{kg} = 3.5 \times 10^6\, M_\odot.$$

- Schwarzschild radius=

$$R_S = \frac{2GM_{BH}}{c^2} = 4.2 \times 10^{-7}\ \mathrm{pc} = 0.08\ \mathrm{AU} = 15 R_\odot.$$

Eddington luminosity

At the Eddington limit

$$L_E = \frac{3GMm_H c}{2r_e^2}$$

where the classical radius of the electron

$$r_e \equiv \frac{e^2}{m_e c^2} = 2.8 \times 10^{-15}\ m$$

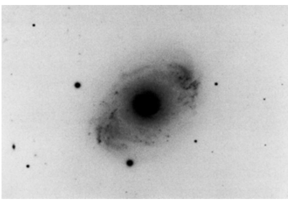
So if L=10^{39} W, and the BH is radiating at the 0.1 x the Eddington limit, then

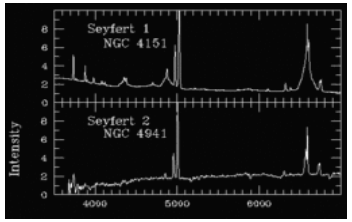
$$M_{BH} = \frac{(2r_e^2)(0.1L)}{3Gm_H c} = 7.7 M_\odot \text{ X } 10^6$$

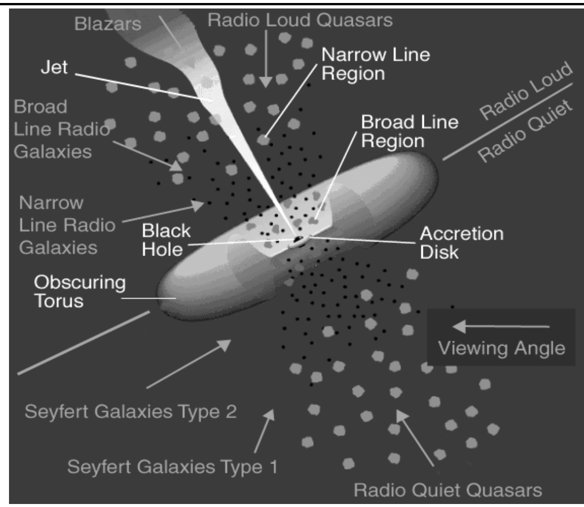
Seyfert I and Seyfert II galaxies

The nuclei of Seyfert I galaxies are bright in the infrared, optical and ultraviolet. They show broad and narrow emission lines.

Seyfert II nuclei are fainter in the UV and the optical but have similar luminosities to Seyfert Is in the IR. Their spectra have only narrow emission lines, not broad ones.







Q10, 2005

(a) Draw a schematic figure showing the major components of the immediate environment of an active galactic nucleus. Label and describe the components you include. Show how the "unified model" helps explain observations of Seyfert I and Seyfert II galaxies and radio-galaxies with lobes with a single model. **[10]**

(b) The Seyfert galaxy NGC 4258 is at a distance of 7 Mpc from us. At the inner edge of the gaseous accretion disk, 0.004 arcsec from the centre, the orbital speed of the gas is 1100 km/s. What is the mass inside this radius? **[6]**

- **Answer:** *The radius of the inner edge of the accretion disk is* $0.004 \times \frac{\pi}{180} \times 7 \times 10^6 = 488\,pc$. *The circular speed is*

$$\frac{v_c^2(r)}{r} = \frac{GM(<r)}{r^2}$$

. *Therefore* $M(<r) = \frac{r}{G} \times (1100 km/s)^2 = 1.4 \times 10^9\, M_\odot$.

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(c) If all of this mass resides in the central black hole, what is its Schwarzschild radius? **[6]**

- **Answer:** *The Schwarzschild radius*

$$R_s = \frac{2GM_{BH}}{c^2} = 3 \times \frac{M_{BH}}{M_\odot} km = 4.2 \times 10^9 \text{ km}$$

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(d) What is the maximum size of the radio-emitting region in a quasar, at a redshift of 3.2, if luminosity variations are observed on a time scale of less than two years? **[8]**

- **Answer:** *The time scale is* $\delta t < 2$ *years. Consider time dilation:* $\delta t(true) = \delta t \times \sqrt{1 - v^2/c^2)}$, *which implies* $v/c = [(1+z)^2 - 1]/[(1+z)^2 + 1] = 0.893$, *where the redshift* $z = 3.2$. *Thus* $\delta t = 2$ *years* $\times \sqrt{1 - (0.893)^2} = 0.9$ *years. The emitting region* $d < \delta t \times c$, *which implies* $d < 0.9 \times 365.25 \times 24 \times 3600 \times 3. \times 10^5$ *km, or* $d < 8.6 \times 10^{12}$ *km* $= 0.3\,pc$.
  [The size-variability argument is commonly available in textbooks, the inclusion of time dilation is a test of the student's ability to make connections with other material.

Gravitational Collapse in the ISM

- The Jeans Mass

$$M_J = \left(\frac{5kT}{G\mu m_H}\right)^{3/2} \left(\frac{3}{4\pi\rho_0}\right)^{1/2}$$

	Diffuse HI Cloud	H_2 Cloud Core
T	50 K	10 K
ρ	5×10^8 m^{-3}	10^{10} m^{-3}
M_J	1500 $M_\odot$	10 $M_\odot$
M_c	1-100 $M_\odot$	10-1000 $M_\odot$

- We know from the ***Jeans Criterion*** that if $M_c > M_J$ collapse occurs.
- Substituting the values from the table into gives:
  - Diffuse HI cloud: $M_J \sim 1500\ M_{Sun}$ => stable as $M_c < M_J$.
  - Molecular cloud core: $M_J \sim 10\ M_{sun}$ => unstable as $M_c > M_J$.
- So deep inside molecular clouds the cores are collapsing to form stars.

Problem of star formation efficiency

Gas in the galaxy should be wildly gravitationally unstable. It should convert all its mass into stars on a free-fall time scale:

$$t_{\rm ff} = \sqrt{\frac{3\pi}{32 G\rho}} = \frac{3.4\times10^{10}}{\sqrt{n}}\ \text{yr}$$

For interstellar medium (ISM) in our galaxy: $n \approx 2\times10^{5}\ \text{m}^{-3}$

$$t_{\rm ff} = 8\times10^{6}\ \text{yr}$$

Total amount of molecular gas in the Galaxy: $\sim 2\times10^{9} M_{\rm sun}$

Expected star formation rate: $\sim 250\ M_{\rm sun}/\text{year}$

Observed star formation rate: $\sim 3\ M_{\rm sun}/\text{year}$

Something slows star formation down...

Calculating star formation rate

- There is an enormous (10^7) **range** in galaxy star formation rates : 10^{-4} - 10^{3} $M_\odot\ yr^{-1}$
  Loosely, we divide this range into two regimes :
  (i) **normal galaxies** (≈75% of local SF) have SFRs : 0 - few $M_\odot\ yr^{-1}$
  note: integrated galaxy spectra ≈varying mix of A-F V (<1 Gyr) and G-K III (3 - 15 Gyr)
  (ii) **starburst galaxies** (≈25% of local SF) range from :
  few $M_\odot\ yr^{-1}$ (SB) → ≈50 $M_\odot\ yr^{-1}$(LIGs) → $10^{2\text{-}3}$ $M_\odot\ yr^{-1}$ (ULIGs)

Question: The Milky Way galaxy has about 5e9 solar masses of gas in total. If 2 solar masses of that gas is turned into stars each year, how many more years could the Milky Way keep up with such a star formation rate?

Answer: 5×10^9 divided by 2 is 2.5×10^9.
So, the MW can keep up its star formation rate of 2 solar masses per year for 2.5×10^9 years.

Abundance and metallicity

[Fe/H]=0 [O/Fe]=0

Notation: =0 means solar abundance.
[ ] denotes logarithmic ratio.

At very early times, [Fe/H]=negative (e.g. =-1.5) since the heavy elements have not be created yet,

Type II Supernovae occur at earlier times. They produce both O and Fe and so [O/Fe] is zero.

At later times, Type I Supernovae make more heavy elements. they contribute to Fe, but not to O. So the ratio [O/Fe] decreases at later times, when the metallicity of stars is higher.

What is a starburst?

There is no well-established definition of the starburst phenomenon but many would agree that in a starburst

- The mean gas consumption timescale is significantly smaller than the Hubble age (as in blue compacts)

  or
- That this is true for a local region in the centre – a *nuclear starburst* (prototype: NGC 7714).

  and that
- Starbursts have *high star formation efficiencies*, i.e they use up a larger proportion of the molecular cloud it is formed from than normal before the cloud disperses or dissociates. The normal efficiency is 5%. In starbursts it may be *10 times higher*.

Global vs. nuclear starbursts

- Global SB affects a substantial part of the of the galaxy and the total gas reservoir is consumed in << Hubble time
- Nuclear starbursts are actually circumnuclear and occur in many massive young clusters

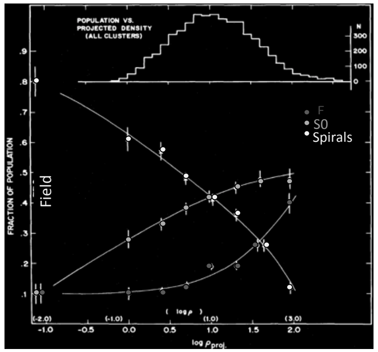
Important questions:

- Is there a fundamental difference in IMF?
- Can starbursts be distinguished from AGNs or other energy sources in the centre?

Nature or Nurture?

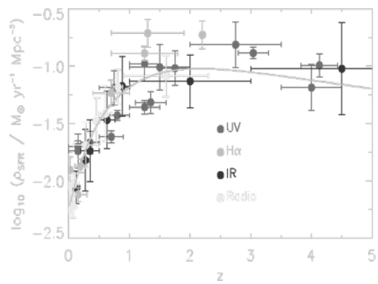
- Nature? Elliptical galaxies only form in protoclusters at high redshift. Rest of population is due to infall.
- or Nurture? Galaxy evolution proceeds along a different path within dense environments.
  - If this is true in groups and clusters, then environment could be the driving force of recent galaxy evolution…

Morphology-Density Relation



Dressler 1980

Why Does Star Formation Stop?



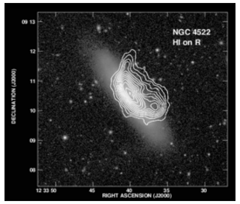
(Hopkins et al 2004)

The "Madau Plot"

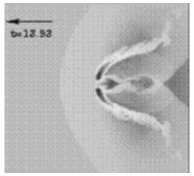
A) Internal? i.e. gas consumption and "normal" aging
B) External? Hierarchical build-up of structure inhibits star formation

Additional physics?

- Ram-pressure stripping
- Collisions / harassment
- "Strangulation"



Kenney et al. 2003

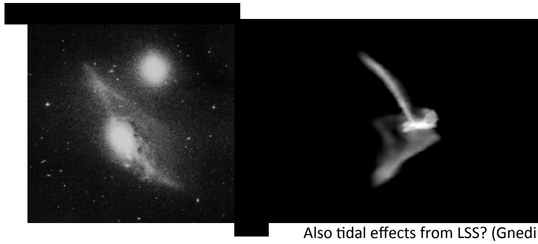


short timescale

Quilis, Moore & Bower 2000

Additional physics?

- Ram-pressure stripping
- Collisions / harassment
- "Strangulation"



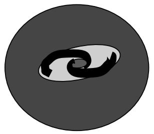
tant in

s?

Also tidal effects from LSS? (Gnedin 2003)

Additional physics?

- Ram-pressure stripping
- Collisions / harassment
- "Strangulation"
  - Either through tidal disruption, or shock-heating to level at which it can't cool (e.g. Springel & Hernquist 2001)



long timescale

When did galaxies form: a rough estimate (1)

Consider a small ($\delta\rho/\rho$ <<1) spherical perturbation of radius r:

$$M(r) \cong (4\pi/3) r^3 \bar{\rho}_{matter} = (4\pi/3) r^3 (3H^2/8\pi G)\Omega_{matter}$$

For H_0=100h km/s/Mpc,

$$M(r)/M_{sun} \cong 1.16\times10^{12} h^2 \Omega_{matter} r_{Mpc}^3$$

For h=0.7 and Ω_{matter}=0.25, $M(r)/M_{sun} \cong 1.4\times10^{11} r_{Mpc}^3$

i.e. an ~L* galaxy coalesced from matter within a comoving volume of roughly ~ 1-1.5 Mpc radius

Jeans mass in an expanding Universe

- The Jeans length depends on the sound speed and density of matter

$$2r \gtrsim \sqrt{\frac{15}{\pi}}\sqrt{\frac{c_s^2}{G\rho}} \approx \lambda_J, \text{ where } \lambda_J \equiv c_s\sqrt{\frac{\pi}{G\rho}}.$$

Early on, while the Universe is radiation-dominated, the density $\rho_r = a_B T^4/c^2$ is low and the pressure is high, with $c_s = c/\sqrt{3}$. So Equation 8.71 gives

$$\lambda_J = c^2\left(\frac{\pi}{3Ga_B T^4}\right)^{1/2} \propto T^{-2}. \quad (8.72)$$

The *Jeans mass* $\mathcal{M}_J$ is the amount of matter in a sphere of diameter λ_J:

$$\mathcal{M}_J \equiv \frac{\pi}{6}\lambda_J^3 \rho_m, \quad (8.73)$$

Jeans mass in an expanding Universe

- The Jeans mass depends on the sound speed and density of matter

$$2r \gtrsim \sqrt{\frac{15}{\pi}}\sqrt{\frac{c_s^2}{G\rho}} \approx \lambda_J, \text{ where } \lambda_J \equiv c_s\sqrt{\frac{\pi}{G\rho}}.$$

- The energy density of an ordinary sound wave is positive, However, the gravitational energy density of a sound wave is negative, since the enhanced attraction in the compressed regions overwhelms the reduced attraction in the dilated regions.
- The Jeans instability sets in wherethe net energy density becomes negative, so that the system can evolve to a lower energy state by allowing the wave to grow, and thus the system to fragment.

Jeans mass: matter dominated

(ii) After recombination, radiation pressure is negligible, so the sound speed

$$c_s^2 = dP/d\rho = \left(\frac{\gamma k_B T}{\mu m_H}\right)^{1/2},$$

where μ is the relative mass of each particle. Thus, the Jeans Mass

$$M_J = \frac{\pi}{6}\rho_m\left(\frac{\pi k_B T}{G\rho_m m_H}\right)^{3/2}.$$

The Jeans mass in the radiation dominated era is of the order of $10^{16} M_\odot$ (scale of galaxy clusters). Adiabatic baryonic perturbations with scale sizes smaller than superclusters cannot grow before recombination.

After recombination, the Jeans mass abruptly falls to globular cluster masses ($\sim 10^5 M_\odot$). At this point, all scales of mass $> 10^5 M_\odot$ become unstable, and pertur-

Jeans mass before decoupling

if we regard the baryons as a minor contaminant, the Jeans length of the photon-baryon fluid was roughly the same as the Jeans length of a pure photon gas:

$$\lambda_J(\text{before}) \approx 3c/H(z_{\text{dec}}) \approx 0.6\,\text{Mpc} \approx 1.9 \times 10^{22}\,\text{m}. \quad (12.27)$$

The *baryonic Jeans mass*, M_J, is defined as the mass of baryons contained within a sphere of radius λ_J;

$$M_J \equiv \rho_{\text{bary}}\left(\frac{4\pi}{3}\lambda_J^3\right). \quad (12.28)$$

Immediately before decoupling, the baryonic Jeans mass was

$$\begin{aligned} M_J(\text{before}) &\approx 5.0 \times 10^{-19}\,\text{kg}\,\text{m}^{-3}\left(\frac{4\pi}{3}\right)(1.9 \times 10^{22}\,\text{m})^3 \\ &\approx 1.3 \times 10^{49}\,\text{kg} \approx 7 \times 10^{18}\,\text{M}_\odot. \quad (12.29) \end{aligned}$$

This is approximately 3×10^4 times greater than the estimated baryonic mass of the Coma cluster, and represents a mass greater than the baryonic mass of even the largest supercluster seen today.

Jeans mass after decoupling

Now consider what happens to the baryonic Jeans mass immediately after decoupling. Once the photons are decoupled, the photons and baryons form two separate gases, instead of a single photon-baryon fluid. The sound speed in the photon gas is

$$c_s(\text{photon}) = c/\sqrt{3} \approx 0.58c\ . \qquad (12.30)$$

The sound speed in the baryonic gas, by contrast, is

$$c_s(\text{baryon}) = \left(\frac{kT}{mc^2}\right)^{1/2} c\ . \qquad (12.31)$$

At the time of decoupling, the thermal energy per particle was $kT_{\text{dec}} \approx 0.26\,\text{eV}$, and the mean rest energy of the atoms in the baryonic gas was $mc^2 = 1.22 m_p c^2 \approx 1140\,\text{MeV}$, taking into account the helium mass fraction of $Y_p = 0.24$. Thus, the sound speed of the baryonic gas immediately after decoupling was

$$c_s(\text{baryon}) \approx \left(\frac{0.26\,\text{eV}}{1140 \times 10^6\,\text{eV}}\right)^{1/2} c \approx 1.5 \times 10^{-5} c\ , \qquad (12.32)$$

only 5 kilometers per second. Thus, once the baryons were decoupled from the photons, their associated Jeans length decreased by a factor

$$F = \frac{c_s(\text{baryon})}{c_s(\text{photon})} \approx \frac{1.5 \times 10^{-5}}{0.58} \approx 2.6 \times 10^{-5}\ . \qquad (12.33)$$

Decoupling causes the baryonic Jeans mass to decrease by a factor $F^3 \approx 1.8 \times 10^{-14}$, plummeting from $M_J(\text{before}) \approx 7 \times 10^{18}\,\text{M}_\odot$ to

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This is comparable to the baryonic mass of the smallest dwarf galaxies known, and is very much smaller than the baryonic mass of our own Galaxy, which is $\sim 10^{11}\,\text{M}_\odot$.

The abrupt decrease of the baryonic Jeans mass at the time of decoupling marks an important epoch in the history of structure formation. Perturbations in the baryon density, from supercluster scales down the the size of the smallest dwarf galaxies, couldn't grow in amplitude until the time of photon decoupling, when the universe had reached the ripe old age of $t_{\text{dec}} \approx 0.35\,\text{Myr}$. After decoupling, the growth of density perturbations in the baryonic component was off and running. The baryonic Jeans mass, already small by cosmological standards at the time of decoupling, dropped still further with time as the universe expanded and the baryonic component cooled.