6. Elliptical galaxies

Even though elliptical galaxies have relatively simple morphologies, and the absence of dust obscuration makes observation simpler, the absence of gas makes it difficult to study their dynamics since radio-telescopes cannot be used. Furthermore, even though the conventional view is that if you've seen one elliptical, you've seen them all, in reality elliptical galaxies can be far more complex than their morphology suggests.

Luminosity profiles

The surface brightness of elliptical galaxies (and bulges of spirals) as a function of radial distance follows the same relation to remarkable accuracy

$$I(R) = I_e\, 10^{-3.33[(R/R_e)^{1/4}-1]} = I_e\, e^{-7.67[(R/R_e)^{1/4}-1]}. \tag{6.1}$$

This is known as the *de Vaucouleurs $R^{1/4}$ law*. The length scale R_e is known as its effective radius, and the numerical value 3.33 is chosen such that if the galaxy were circularly symmetric, then half the total light of the galaxy would lie within R_e. The outer parts of certain giant ellipticals, normally found at the centre of rich clusters of galaxies, show more light than is expected from the de Vaucouleurs profile– these are known as cD galaxies (see, e.g., B&M Fig. 4.28).

Problem 6.1: Show that the total luminosity of a galaxy with the $R^{1/4}$ profile (6.1) is $L_{\rm tot} \sim 7.2\pi R_e^2 I_e$. [You might need to use the following integral: $\int_0^\infty t^7\, e^{-t} = \Gamma(8) = 7!$].

Problem 6.2: Show that the central surface brightness of an elliptical galaxy with an $R^{1/4}$ profile (6.1) is $I_0 \sim 2000\, I_e$, and its mean surface brightness within radius R_e is $\sim 3.6\, I_e$.

The Fundamental Plane

The luminosity of an elliptical roughly scales as its average velocity dispersion as the *Faber-Jackson relation*

$$L \propto \sigma^4, \tag{6.2}$$

and is often used to measure distances to ellipticals (this was the relation used by the 'Seven Samurai', for instance, in the study that found evidence for a 'Great attractor' in our neighbourhood). But it turns out that all ellipticals don't obey the F-J relation in the same way–the surface brightness of the elliptical plays a role as well.

If we assume that the velocity dispersion of stars σ and the M/L ratio is constant throughout an elliptical galaxy, we can use the virial theorem to infer a relation between the global measurable parameters of ellipticals. From the virial theorem we have $2T + V = 0$, or, approximately,

$$Mv^2 - \frac{3}{5}\frac{GM^2}{R} = 0.$$

This gives mass $M \sim v^2 R/G$. The *mass surface density* then should go as

$$\Sigma \equiv M/R^2 \sim \frac{v^2}{GR},$$

whereas the *surface brightness* is

$$I \equiv L/R^2 = \frac{L}{M}\frac{M}{R^2} \sim \frac{v^2}{GR}.\frac{1}{M/L}.$$

For an elliptical galaxy which has very little rotation, the v in the above equation is really its velocity dispersion σ. Replacing the R by the characteristic half-light radius R_e, we have

$$R_e \sim \sigma^2\, I^{-1},$$

whereas the measured result from real ellipticals is (see S&G, Fig. 6.13)

$$R_e \sim \sigma^{1.24}\, I^{-0.82}.$$

This is known as the *Fundamental Plane* relation, and the fact that the theoretical expectation doesn't match the observed relation probably shows that our assumption of the M/L ratio being constant throughout the galaxy isn't correct.

PROBLEM 6.3: Show that if all ellipticals had the same surface brightness I_e at radius R_e, they would follow the Faber-Jackson relation (6.2).

DO ELLIPTICAL GALAXIES ROTATE?

The tensor virial theorem, which can be derived from the collisionless Boltzmann equation (see B&T §4.3) predicts that if the flattened shape of an elliptical galaxy were due to its rotation, then the ratio of its average rotational speed to its average velocity dispersion would be

$$\frac{v}{\sigma} = \sqrt{\frac{\epsilon}{1-\epsilon}},$$

where $\epsilon = 1 - b/a$ is the ellipticity of the galaxy. This applies to an isotropic rotating oblate spheroid with similar, concentric density contours– a somewhat idealized, but realistic model of an elliptical galaxy. According to this relation, even fairly round galaxies ($b/a \sim 0.7$) should rotate fairly fast.

Observations of ellipticals, however, indicate that luminous elliptical galaxies span a range of values for v/σ, but all these values are far too small to indicate that elliptical galaxies are flattened by rotation (see S&G, Fig. 6.14 or B&T Fig. 4.6). The flattening in these systems is caused by velocity anisotropy of their stars. Fainter ellipticals, and bulges of spirals, however, have $v/\sigma \sim 1$, indicating a significant role for rotation in determining their shapes. These are probably composite systems, with a fast-rotating stellar disk embedded within a slower-rotating ellipsoidal outer galaxy.