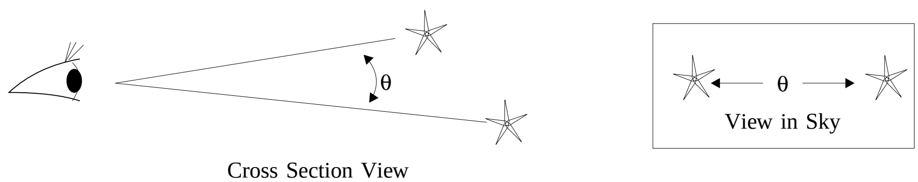
2 The Astronomical Context

Here we learn more about available types of astronomical data, in order from easy to difficult.

2.1 Angular and positional measurements

2.1.1 Angles between objects measured "on the sky" – i.e., in projection



We can usually measure small angles ($< 1°$) much better than large angles, (say, $45°$). We find accuracies of:

Optical	$\Delta\theta \sim 0.''001$	best
Radio	$\Delta\theta \sim 0.''000001$	best (Very-Long Baseline Interferometry (VLBI))
X-ray	$\Delta\theta \sim 2''$	limited by spacecraft
γ-ray	$\Delta\theta \sim 1°$	limited by properties of γ-ray detectors

Useful conversions:

- $1° = 60' = 3600''$
- 1 radian $= 57.3° = 206,000''$

- 1 mrad = 3.5′

- 1 μrad = 0.21″

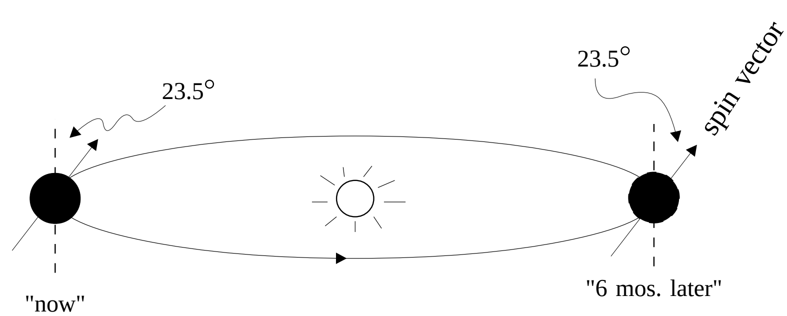
2.1.2 Coordinate systems in the sky

Positions of objects in the sky can be given in various coordinate systems.

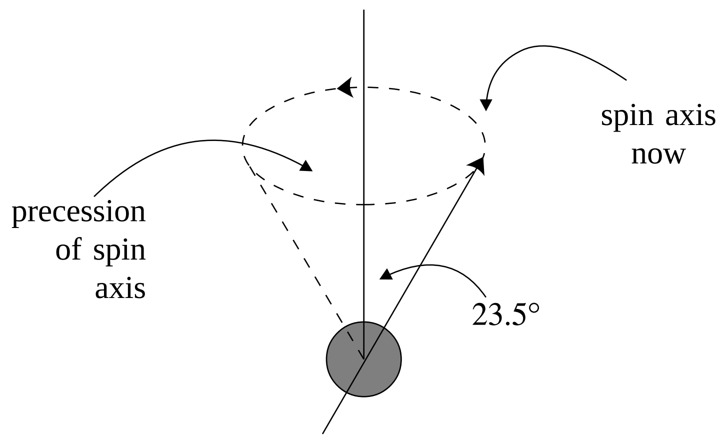
Equatorial Celestial Coordinates (α, δ) or (RA, DEC)

The Earth is a pretty good gyroscope, so its axis points a constant direction in inertial space: the North Celestial Pole (NCP). Declination (δ or DEC) is measured from $+90°$ at that pole to $-90°$ at the South Celestial Pole. The longitude-like coordinate is called "right ascension" (α or RA) and is measured (confusingly) in *hours* (24h= 360°). The zero point is at the "vernal equinox" (where the Sun is in the sky at the beginning of Spring, also called the "first point of Ares"). RA increases in number in the direction the sky moves (as the Earth turns). That is, a fixed telescope sees increasing RA positions with time (1 hour RA per hour of sidereal time).

One peculiarity is that objects at rest on sky have RA/Dec which vary very gradually with time because of the Earth's precession. This occurs because the spin axis of the earth is not aligned with the Earth-Sun orbital plane.



If the earth was a sphere, this would not affect the Earth's spin axis (there would be no coupling), but tides and rotation distort the earth, so it feels a net torque from the gravity of the sun/moon. This torque makes the polar axis (direction to NCP) precess with a period of 26,000 years.



Every 26,000 years, the RA goes through a big loop, and DEC changes by $\pm 23.5°$. Positions of objects in the sky change by tens of arcseconds per year. This is easily detectable, as we can measure the angle to 10^{-3} arcsec.

Coordinates quoted for objects are therefore refered to a particular date or "standard epoch" to remove effects of precession. Standards are

- B1950 (going out of use)
- J2000 (coming in)

Here the "B" and "J" refer to technicalities of the model, while "1950" and "2000" are the reference dates (typically noon on January 1 of the date).

Galactic Coordinates (l, b)

The equator of the galactic coordinate system is the galactic plane (the Milky Way). l, the longitude coordinate, is zero in the direction of the Galactic Center. b, the galactic latitude goes from $+90°$ at the North Galactic Pole (NGP) down to $-90°$ at the SGP.

2.1.3 Angular separations from coordinates in Sky

First convert from latitude-longitude style coordinates to Cartesian unit vectors, e.g.,

$$\hat{r}_z = z = \sin b$$

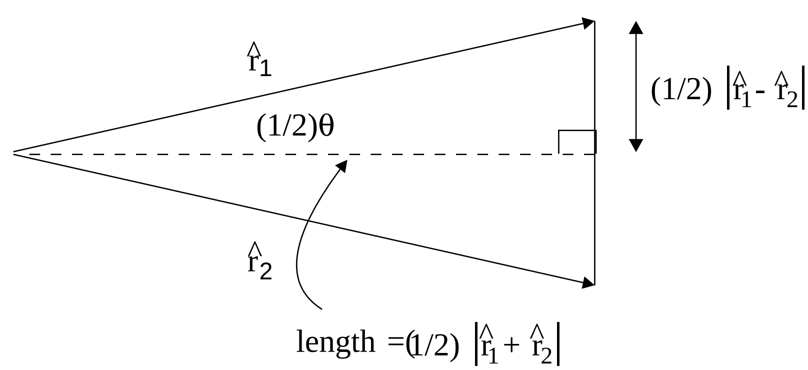
$$\hat{r}_x = x = \cos b \cos l$$

$$\hat{r}_y = y = \cos b \sin l$$

Then use vector formulas to get the angle between the two unit vectors.

$$\cos\theta = \hat{r}_1 \cdot \hat{r}_2$$

is ok when θ is *not small*, but for small separations it becomes inaccurate (angles pile up around $\cos\theta = 1$). In that case use the construction



which yields

$$\tan\frac{1}{2}\theta = \frac{|\hat{r}_1 - \hat{r}_2|}{|\hat{r}_1 + \hat{r}_2|} .$$

2.1.4 Solid angles

When observing an object from a point, the solid angle subtended by the object refers to the fraction of all "lines of sight" that the object covers. In analogy with the definition of radians, the unit of solid angle, the *steradian*, measures the area on the unit sphere that the object covers. Since a sphere has area $4\pi r^2$ there are 4π *steradians* in the whole sky.

The corresponding number of square degrees is

$$4\pi \left(\frac{360}{2\pi}\right)^2 = 41252.96 \approx 40000$$

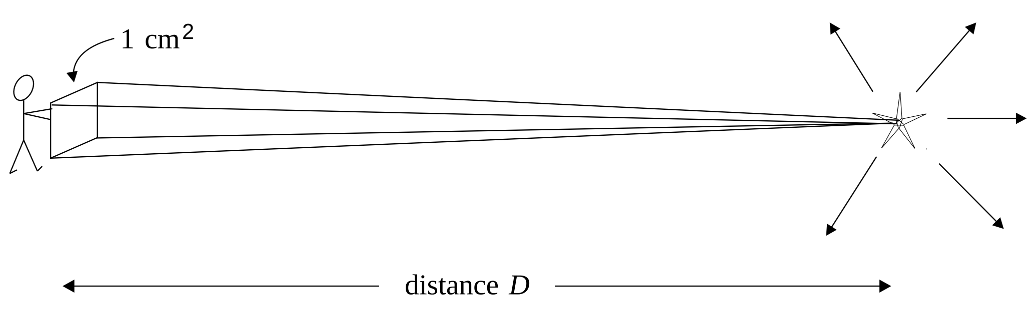
since $\left(\frac{360}{2\pi}\right)$ is the "radius of a unit circle in degrees."

The Sun and Moon are both very close to $0.5°$ diameter, so their areas are each $\pi(0.5)^2/4 = 0.20$ square degrees, or $\approx 1/200000$ of the celestial sphere. (If you think hard, you will see why this tells you that full moonlight is about 10^5 times dimmer than full sunlight!)

2.2 Brightness measurements

2.2.1 Flux and UBV system

Flux is the energy arriving from a particular object (star, galaxy,...) per unit area of detector per unit time, so it has units erg $\mathrm{cm^{-2}s^{-1}}$.

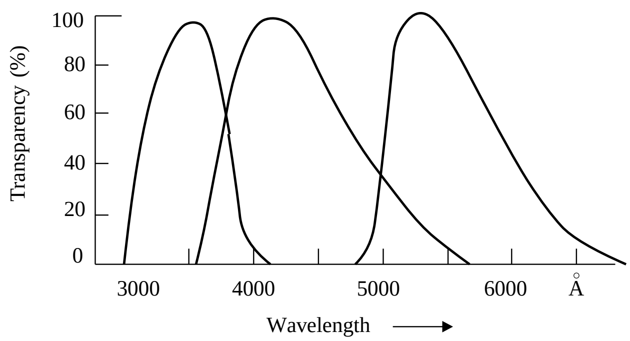


You can see that if the source emits isotropically (same in all directions), the relation between flux F and total luminosity L (erg $\mathrm{s^{-1}}$) is

$$F = \frac{L}{4\pi D^2} \, .$$

Ideally, Flux would be the *total* energy emitted, but in practice we measure different wavelength bands with different instruments or detectors. In such cases we label it by a letter telling which band (e.g. V = visual) and it then means "energy per area per time arriving *in that band*."

The most common bands are U = "ultraviolet," B = "blue," V = "visual," R = "red," I, J, K = "infrared."



Light Transmission Through UBV Filters. This graph shows the wavelength ranges over which the standardized U, B, and V filters are transparent to light. The U filter is transparent to the near-ultraviolet. The B filter is transparent from about 380 to 550 nm, and the V filter is transparent from about 500 to 650 nm.

Band	U	B	V	R	I	J	K
Central Wavelength	3650Å	4400Å	5500Å	7000 Å	9000Å	1.25μ	2.2μ
Effective width (defined (by area under curve)	680Å	980Å	890Å	2200Å	2400Å	0.38μ	.48μ
$\log f_\lambda$ (erg cm^{-2} s$^{-1}\mu^{-1}$) for $m = 0$	-4.37	-4.18	-4.42	-4.76	-5.08	-5.48	-6.40

Note that 1000Å= 100 nm = 0.1μ. We write F_U, F_B, F_V, etc. for in-band fluxes. When we need a word for "real, total energy" we say *bolometric*, so:

$$L_{\rm bol} = \text{ total luminosity in all bands.}$$

To interpret the last line in the above table, we need to know about the magnitude scale.

2.2.2 Apparent magnitude

Apparent magnitude is defined logarithmically (property of eye) such that an *increase* in magnitude by 5 corresponds to a factor of 100 in apparent brightness (flux). Thus, if we have two stars with fluxes F_1 and F_2, their magnitudes obey

$$m_2 - m_1 = -2.5 \log_{10} \left(\frac{F_2}{F_1} \right) \ .$$

This can be written

$$m_2 + 2.5 \log F_2 = m_1 + 2.5 \log F_1 \equiv \text{constant.}$$

Once we decide on magnitude for *one* star all others are determined.

Since stars have different colors (we'll learn why later) we must compare them in a specific color band (i.e. look at them through filters as shown in 2.2.1). The naked eye is esentially a V filter. Thus, like fluxes, magnitudes are written with a subscript indicating band: $m_{\mathrm{V}}, m_{\mathrm{B}}$, etc. Sometimes this is written just V, B, etc.

The zero point constant derives historically from the ancient Greeks who named some bright stars as being "of the first magnitude" (what we would now call V= 0):

Arcturus (α Boo)	V= -0.06	$\mathrm{B} - \mathrm{V} = 1.23$
Vega (α Lyr)	V = 0.04	$\mathrm{B} - \mathrm{V} = 0.0$
Capella (α Aur)	V = 0.8	$\mathrm{B} - \mathrm{V} = 0.79$

Note that larger magnitude means "dimmer":

Betelgeuse (α Ori)	V = 0.8	$\mathrm{B} - \mathrm{V} = 1.85$
Aldebaran (α Tau)	V = 0.85	$\mathrm{B} - \mathrm{V} = 1.53$

Larger "color difference" means redder color. With good eyes and a dark sky (not Cambridge!) you can see stars down to V $\approx$ 6.

How much energy do we receive from Betelgeuse in the V band?

From the previous table, for V=0

$$F_{\rm V} = 10^{-4.42} \frac{\rm erg}{\rm cm^2\, s\mu} \times 0.089\mu = 3.4 \times 10^{-6} \frac{\rm erg}{\rm cm^2\, s} \ .$$

But Betelgeuse has V=0.8, so its

$$F_{\rm V} = (3.4 \times 10^{-6}) 10^{-0.4(0.8)} = 1.6 \times 10^{-6} \frac{\rm erg}{\rm cm^2\, s}$$

You can see that the general relation for any band X (e.g. $X = {\rm U, B, V} \ldots$)

$$F_X = 10^{-C_X} W_X 10^{-0.4 m_X}$$

where C_X is the band's $\log f_\lambda$ for $m_X = 0$, W_X is its effective width, and m_X is the apparent magnitude of the object in question.

Incidentally, since $10^{-0.4} = 0.398$ is pretty close to $e^{-1} = 0.367$, magnitudes are not too different from e-folds. This is useful for calculating small magnitude differences in your head, e.g.

$$0.03 \text{ mag} \approx e^{0.03} \approx 1 + 0.03 = 1.03 \ .$$

So a difference of 0.03 mag is about a 3% flux difference, and so forth.

Another useful fact if you like decibels is that 1 mag = 4 dB. Therefore, e.g. 2.5 mag = 10 dB = factor of 10 in intensity.

2.2.3 Absolute Magnitude

Absolute magnitude, denoted $M_{\rm B}, M_{\rm V}$, etc ., is defined as the magnitude an object *would have* if it were 10 pc away. Thus, since $F \propto D^{-2}$ (D the distance)

$$m = M + 5 \log \left(\frac{D}{10 \text{ pc}} \right) = M + 5 \log D - 5$$

(the 2.5 on log flux becomes a 5 on log distance). The quantity

$$m - M = 5 \log D - 5$$

is called the distance modulus.

You should now be able to derive the relation between absolute magnitude M and physical luminosity L (in a given color band) for an object. These are properties of the object, not of its distance.

If we know L (or M) for an object then measurement of m gives D, the distance.

A "standard candle" is a hoped-for class of objects which has a luminosity (absolute mag) which can be determined easily without knowing its distance.

Notice that color differences (e.g. B − V) are independent of distance and are equal to $M_{\text{B}} - M_{\text{V}}$, e.g.

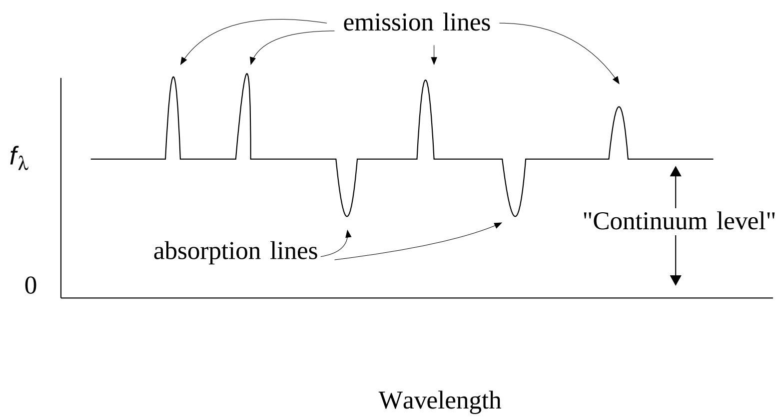
Here are the masses, absolute magnitudes, and (so-called) spectral types of stars on the "main sequence" (we'll learn more about this later).

Spectral type:	O	B	A	F	G	K	M
Typical Mass (units of $M_\odot$)	40	6	2	1.5	1.0	0.7	0.3
M_{V}	−5.8	−1.1	2.0	3.4	5.1	7.3	11.8

The Sun is a G star; at 10 pc it would be barely visible, $m_{\text{V}} = 5.1$.

2.2.4 Spectra

Atoms in a gas can emit light at specified frequencies (called spectral lines). They can also absorb light at these frequencies. Whether they on balance emit or absorb depends on their temperature, ionization, density, etc. So, a spectrum might look like:

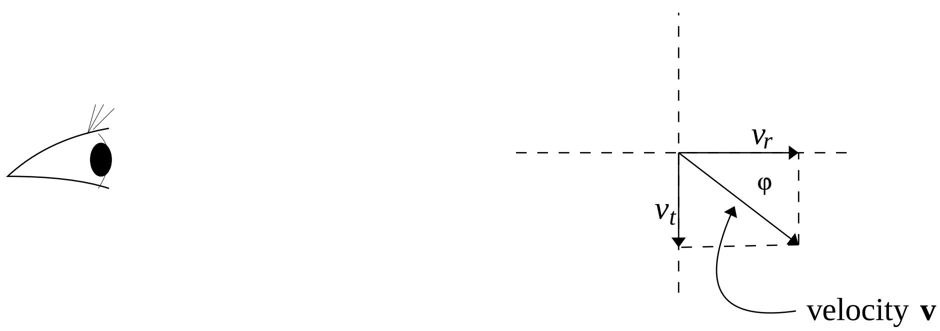


The units of f_λ are ergs per cm^2 (collecting area) per s *per wavelength.* (This is the same f_λ that we previously saw in the UVB magnitude table).

2.3 Velocity measurements

Velocity comes in two flavors:

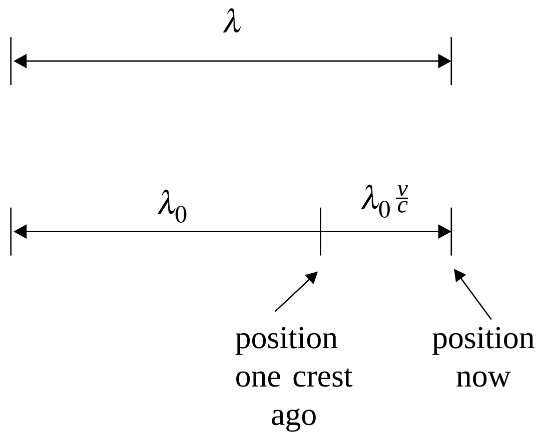
i) radial velocity: measured by Doppler shifts

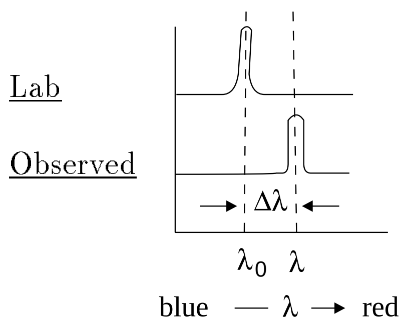


$$\frac{\Delta\lambda}{\lambda_o} = \frac{\lambda - \lambda_o}{\lambda_o} = z = \frac{v_r}{c} \quad \text{in nonrelativistic limit}$$

Between successive emissions of "crests" (time intervals λ_0/c) the emitter moves a distance $v/(\lambda_0/c)$, while the wave moves a distance λ_0, so

$$\lambda = \lambda_0 + \lambda_0 \frac{v}{c}$$
$$\Rightarrow \frac{\lambda - \lambda_0}{\lambda_0} = \frac{v}{c}$$





Redshift $\equiv z > 0$
$\equiv$ object receding

Blueshift $\equiv z < 0$
$\equiv$ object approaching

ii) proper motion

Here we observe motion of object in plane of the sky over time (sometimes a very long time). We get $v_t = v \sin\phi$ from the rate of change of position on the sky *if we know the distance to the object.*

We generally *do* know the distance (from parallax measurements q.v.) because we can only see proper motions for nearby stars.

E.g., to see appreciable motion, say $\sim 1''$/century, given that the typical velocity of a star is ~ 10 km s^{-1}, we can only measure v_t for stars within distances $\lesssim 2$ kpc, about one-fifth the distance to the center of the galaxy.

We can do better with VLBI radio techniques using masers (an astronomical analog to a laser) because (1) we can determine angles more accurately, and (2) maser velocities are larger than velocities of nearby stars. This has been used to get the distance to the galactic center.

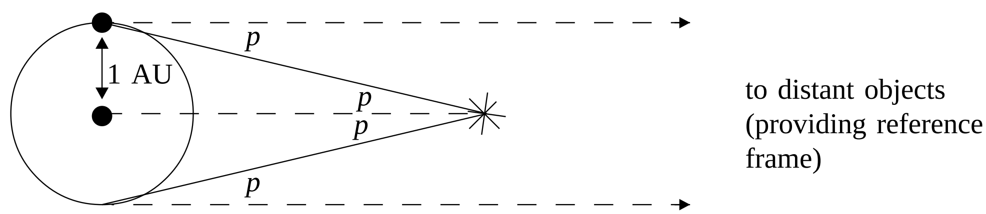
2.4 Distance measurements

The difficult one. Problems measuring distances cause many arguments, e.g., the Hubble constant in cosmology. Various methods are used.

i) radar: round-time measurements (Venus, Moon). Okay for nearby planets; used to set the scale of the solar system: 1 AU $= 1.50 \times 10^{11}$ m $= 1.50\times 10^{13}$ cm.

ii) triangulation: again okay for Venus, Moon. Taken with radar, it leads to measurements of the AU.

iii) parallax: (using Earth's orbit)



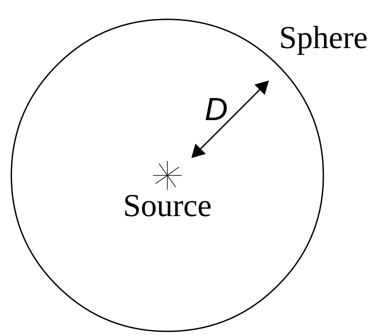
Angles p are small ($< 1''$) and the distance $D = R/p$. If $R = 1$ AU, and p is measured in seconds of arc, then D comes out in parsecs. 1 pc = distance at which 1 AU subtends 1 arcsec = 3.086×10^{18} cm.

iv) inverse square law and "standard candles"

If you know that a certain object has luminosity L (e.g., by looking at similar objects near you) [L in erg/sec or watts], then the distance of that object is given from its observed flux

$$F = \frac{L}{4\pi D^2} \quad .$$

(Remember F is in energy/sec/unit area.)



This method works well for Cepheids (variable stars whose variability period, obtained from long-time observations, is related to L. Thus, we use the measured period to infer L and combine L with the observed brightness to get D). RR Lyrae stars (another kind of variable star) and Tully-Fisher galaxies (in which the galaxy's luminosity is inferred from its rotational velocity) are used in a similar way. Note that extinction, the absorption of light by intervening material, can cause problems.

v) angular sizes of "standard rods"

Same idea as iv): find a distant object that is the same as a nearby one for which you know the size. If the nearby object has a true size L, and the distant object has angular size θ, then

$$D = \frac{L}{\theta} \quad .$$

vi) Hubble law (galaxies and beyond)

Use the concept that the Universe is expanding. *If* the Hubble law has been calibrated for you (this involves getting measurements of distances of objects by independent means), then a radial velocity measurement leads to the distance from

$$v_r = H_o D \quad ,$$

where v_r is the Doppler velocity, and $H_o = (50\text{–}100)$ km s^{-1} pc^{-1}. This is usually written $H_o = 100\ h$ km s^{-1} pc^{-1}, where $h = (0.5\text{–}1.0)$. But note that peculiar velocities and "gravitational redshifts" might cause problems.

Note that the use of method vi) gives an uncertainty of a factor ~ 2 in the distance scale! This is a real problem for astrophysics. H_o is not well known because of the difficulty in determining distances to distant galaxies from methods iv) and v).