Fluid dynamics in neutron star oceans

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Outline

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Low mass X-ray binaries

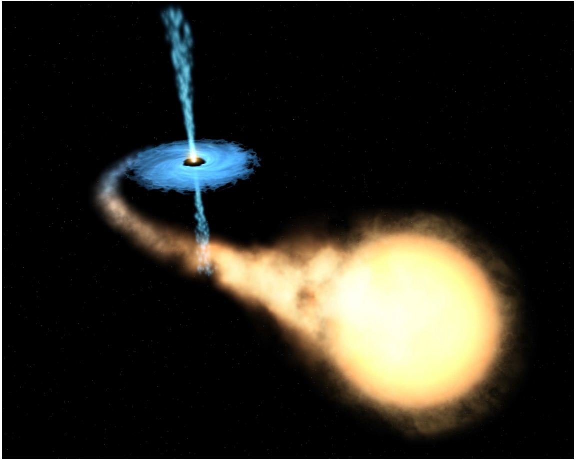

Figure : NASA/ESA

Type I X-ray bursts

- Neutron stars in low mass X-ray binaries **accrete** matter from companion
- Density and temperature in ocean eventually reach point where ignition occurs $\rightarrow$ **Type I X-ray burst**
- Bursts typically occur every **few hours to days**
- Understanding bursts will help put tighter limits on other NS properties, e.g. radius, magnetic field strength

Type I X-ray bursts
Observed light curve

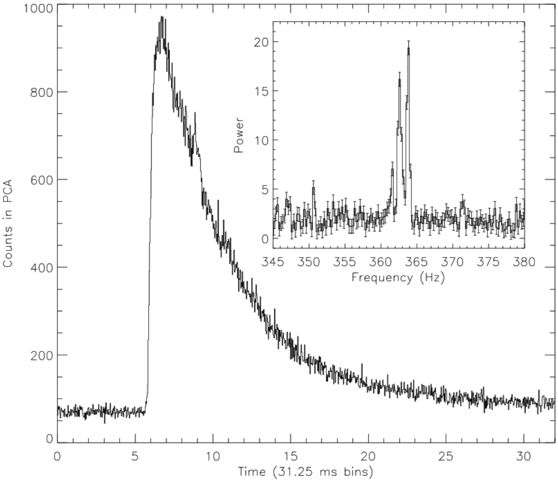

Figure : Burst from 4U 1728–34 (Strohmayer et al. 1996)

Type I X-ray bursts

Flame propagation

- Mechanisms (potentially) influencing flame propagation:
  - Coriolis force
  - ocean composition
  - non-radial oscillations
  - magnetic fields
  - crustal interface waves
  - latitude of ignition
  - oblateness
  - fast rotation
  - **general relativity**
- At NS surface, $|\Phi|/c^2 \sim 0.1$, so GR likely to be important

Low Mach approximation

Why?

- Numerical models limited by CFL (Courant–Friedrichs–Lewy) limit:

$$\frac{v\Delta t}{\Delta x} \leq C_{\mathrm{max}}$$

- Models by Cavecchi 2013 & Spitkovsky et al. 2002: flames propagate with $v \sim 10^5\,\mathrm{cm\,s^{-1}} \Rightarrow M = v/c_s \sim 10^{-3} \ll 1$
- Can evolve models using v rather than sound speed $\rightarrow$ use **much larger time steps**

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Low Mach approximation
How?

- Decompose thermodynamic variables e.g. pressure as

$$p(\vec{x}, r, t) = p_0(r, t) + \pi(\vec{x}, r, t),$$

  where $\pi / p_0 = O(M^2)$
- Enforce conservation of energy using constraint term in Euler equation, equation of state cast as velocity constraint

Low Mach approximation
Testing the equations

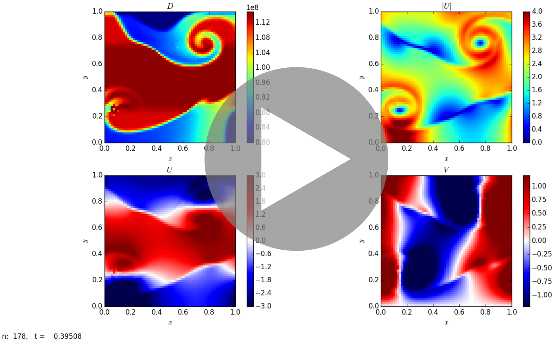

Figure : Vortex simulation can be found at
`http://www.southampton.ac.uk/~ah1e14/`

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Conclusions and future plans

- Modelling **propagation of burning fronts** in NS oceans
- Including **GR**, plan to include fast rotation & oblateness
- Model burning front as a level set
- Collaborate with MAESTRO team - shall incorporate relativistic equations in their code
- Future for Low Mach: binary inspiral?

Thank you for listening

Relativistic fluid equations and Wilson formulation

- General relativistic fluid equations:

$$\begin{aligned} \nabla_{\mu}\left(\rho u^{\mu}\right) &= 0 \\ u^{\mu}\nabla_{\mu}\left(\rho h - p\right) + \rho h \nabla_{\mu} u^{\mu} &= 0 \\ \rho h u^{\nu}\partial_{\nu} u_{\mu} + \partial_{\mu} p + u_{\mu} u^{\nu}\partial_{\nu} p &= \rho h \Gamma_{\rho\nu\mu} u^{\nu} u^{\rho} \end{aligned}$$

- Wilson formulation: $D = \rho u^0$ and $U^{\mu} = u^{\mu}/u^0$.

Low Mach number equations

Continuity	$\partial_t D + \partial_i (D U^i) = -D \Gamma^{\mu}{}_{\mu\nu} U^{\nu}$
Energy	$\partial_t (Dh) + \partial_i (U^i D h) = u^0 \frac{D p_0}{Dt} - Dh \Gamma^{\mu}{}_{\mu\nu} U^{\nu}$
Momentum	$\partial_t U_j + U^i \partial_i U_j = -U_j \frac{D \ln u^0}{Dt} + \Gamma_{\rho\nu j} U^{\nu} U^{\rho} - \frac{1}{D h u^0} \left(\partial_j p_0 + \xi \partial_j \left[\frac{\pi}{\xi} \right] \right)$
Velocity constraint	$-\nabla_{\nu} \pi + \frac{\pi}{2\sqrt{-g}\kappa \Gamma_1 p_0} \nabla_{\nu} p_0 = 0$
Integrating factor	$\xi = A \exp \left(\frac{\ln p_0}{2\sqrt{-g}\kappa \Gamma_1} \right)$

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