



Constraints on the gravitational wave background with pulsar timing arrays

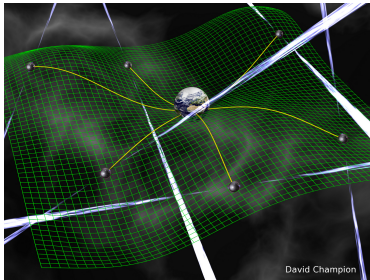
Hannah Middleton, Siyuan Chen, Walter Del Pozzo,
Alberto Sesana, Alberto Vecchio
hannahm@star.sr.bham.ac.uk

Cosmic Mergers Workshop – 22 September 2017

Overview

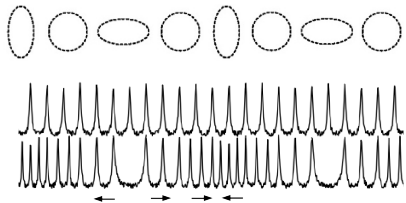
- Pulsar timing searches for gravitational wave background
- No detection yet, but upper limits reaching astrophysically interesting sensitivities
- Recent doubt cast on binary assembly theories
→ are mergers stalling / accelerated??
- Bayesian analysis with astrophysical prior
- Are prediction consistent with upper limits?
→ yes so far!

Pulsar timing arrays

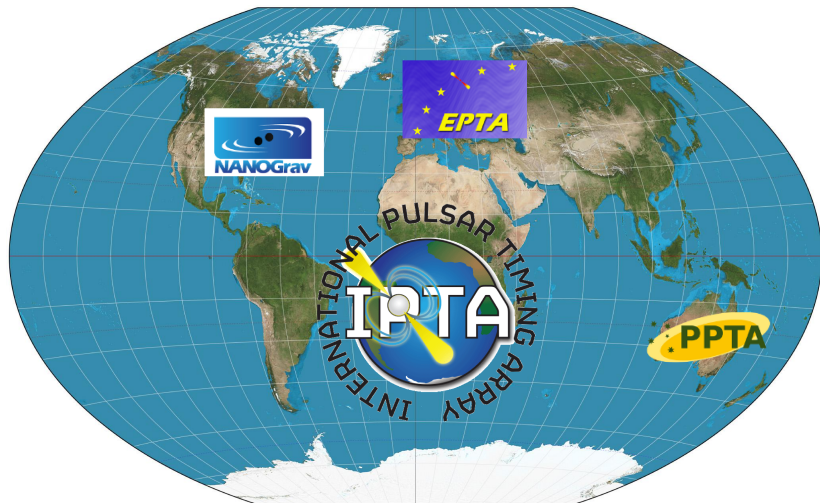


Pulsar timing array

- Gravitational wave background from many mergers
- At nHz frequencies
- Search for deviations in pulse time of arrivals



Searching for the background



Recent results – data

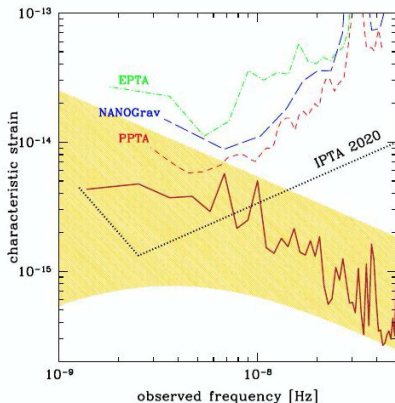


Image: A. Sesana
(reproduced from Hobbs & Dai 2017)

- Most stringent upper limit from Parkes Pulsar Timing array (Shannon et al 2015)
- $h_{ul} < 1 \times 10^{-15}$ at 95% confidence ($f = 1/1\text{yr}$)
- Are predictions in trouble?
 - Eccentricity?
 - Stalling?

Can we place any constraints on the population?

- Bayesian analysis with astrophysical prior
- Can we make any statements on our current predictions for the gravitational wave background
- Our model:
 - merger rate density
 - chirp mass distribution ($\mathcal{M} = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$)
 - redshift distribution
 - eccentricity at decoupling from the environment

Model

$$h^2(f) = \frac{4G}{\pi c^2 f} \int_0^\infty dz \int_0^\infty d\mathcal{M} \frac{d^2 n}{dz d\mathcal{M}} \frac{dE}{df_r} e_t$$

observed GW frequency

sum sources over redshift & chirp mass

number of sources per redshift and chirp mass interval

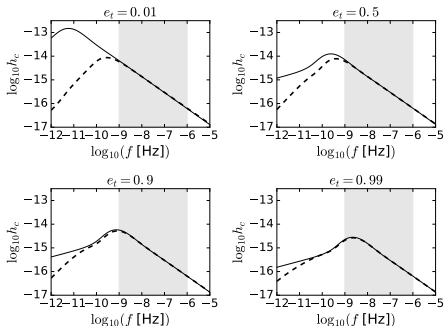
energy spectrum for eccentric binaries

emitted GW frequency (source frame)

e_t

Chen, Sesana & Del Pozzo 2017 (10.1093/mnras/stx1093)

Eccentricity



- Some eccentricity at decoupling
- Population of eccentric gravitational wave driven binaries
- Environmental influence affects lower than PTA band
- Depletes sources at low frequency
- This is the **same** for all binaries

Chen, Sesana & Del Pozzo 2017 (10.1093/mnras/stx1093)

Model

$$h^2(f) = \frac{4G}{\pi c^2 f} \int_0^\infty dz \int_0^\infty d\mathcal{M} \left(\frac{d^2 n}{dz d\mathcal{M}} \right) \frac{dE}{df_r} e_t$$

The diagram illustrates the components of the equation for the gravitational wave energy spectrum $h^2(f)$. The equation is written in blue, with the term $\frac{d^2 n}{dz d\mathcal{M}}$ circled in green. Green arrows point from descriptive text labels to the corresponding parts of the equation:

- observed GW frequency** points to f in the denominator of the prefactor.
- sum sources over redshift & chirp mass** points to the double integral $\int_0^\infty dz \int_0^\infty d\mathcal{M}$.
- number of sources per redshift and chirp mass interval** points to the circled term $\frac{d^2 n}{dz d\mathcal{M}}$.
- energy spectrum for eccentric binaries** points to $\frac{dE}{df_r}$.
- emitted GW frequency (source frame)** points to f_r in the denominator of $\frac{dE}{df_r}$.
- e_t** points to the eccentricity parameter e_t .

Model

$$\frac{d^2 n}{dz d\mathcal{M}} = \dot{n}_0 \left[\left(\frac{\mathcal{M}}{10^7 M_\odot} \right)^\alpha \exp \left(-\frac{\mathcal{M}}{\mathcal{M}_*} \right) \right] \left[(1+z)^\beta \exp \left(-\frac{z}{z_*} \right) \right] \frac{dt_r}{dz}$$

chirp mass distribution

redshift distribution

source frame time

no. mergers ($\text{Mpc}^{-3} \text{Gyr}^{-1}$)

AND e_t

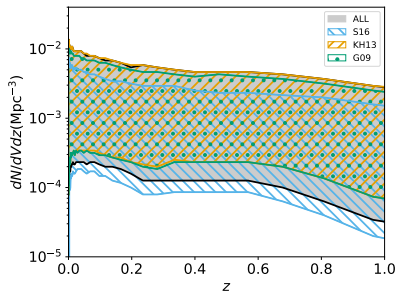
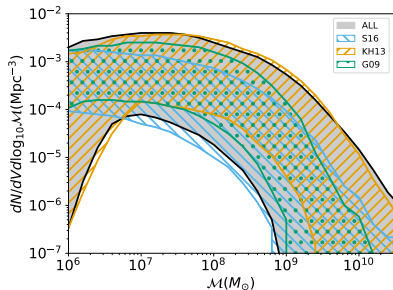
6 parameters model:

- \dot{n}_0 (merger rate density)
- α, \mathcal{M}_* (chirp mass distribution)
- β, z_* (redshift distribution)
- e_t (decoupling eccentricity)

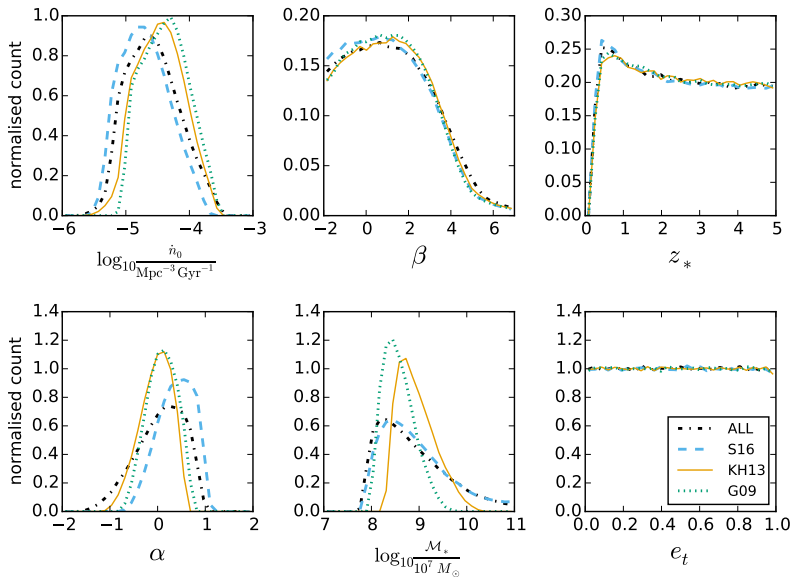
Astrophysical Prior

| Model | median strain at $f = 1/1\text{yr}$ |
|-----------------------|--|
| Pessimistic \approx | 4×10^{-16} |
| Middling \approx | 7×10^{-16} |
| Optimistic \approx | 1.5×10^{-15} |
| All+ \approx | 8×10^{-16} |

(Shankar+ 16, Gültekin+ 09,
Kormendy Ho 13)



Astrophysical Prior



Chen, Middleton, et al. doi.org/10.1093/mnras/stx475
Middleton, Chen, et al. [arXiv:1707.00623](https://arxiv.org/abs/1707.00623)

Method Overview

$$p(\theta|dM) = \frac{p(\theta|M)p(d|M, \theta)}{p(d|M)}$$

Method Overview

PRIOR
for 6 parameters



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Method Overview

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LIKELIHOOD:
upper limit or
detection?

$$p(\theta|dM) = \frac{p(\theta|M)p(d|M, \theta)}{p(d|M)}$$

Method Overview

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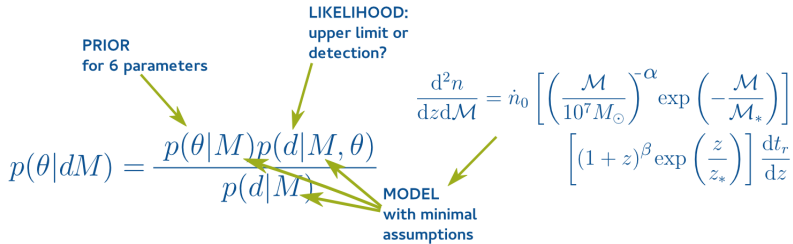
MODEL
with minimal
assumptions

Method Overview

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LIKELIHOOD:
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MODEL
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$$p(\theta|dM) = \frac{p(\theta|M)p(d|M, \theta)}{p(d|M)}$$
$$\frac{d^2n}{dzd\mathcal{M}} = \dot{n}_0 \left[\left(\frac{\mathcal{M}}{10^7 M_\odot} \right)^{-\alpha} \exp \left(-\frac{\mathcal{M}}{\mathcal{M}_*} \right) \right] \left[(1+z)^\beta \exp \left(\frac{z}{z_*} \right) \right] \frac{dt_r}{dz}$$


Method Overview

PRIOR
for 6 parameters

LIKELIHOOD:
upper limit or
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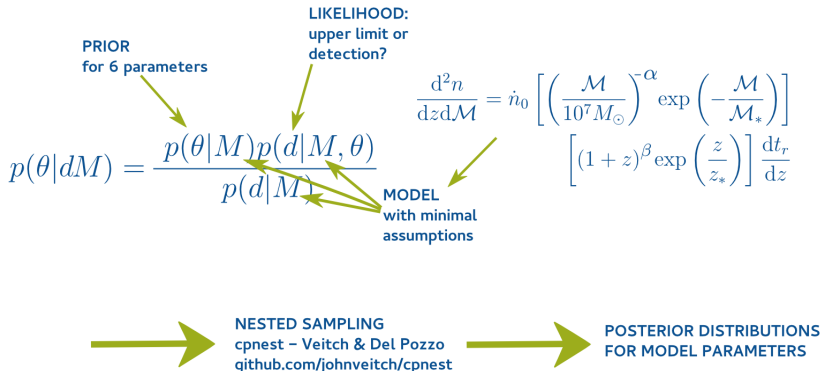
MODEL
with minimal
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$$p(\theta|dM) = \frac{p(\theta|M)p(d|M, \theta)}{p(d|M)}$$
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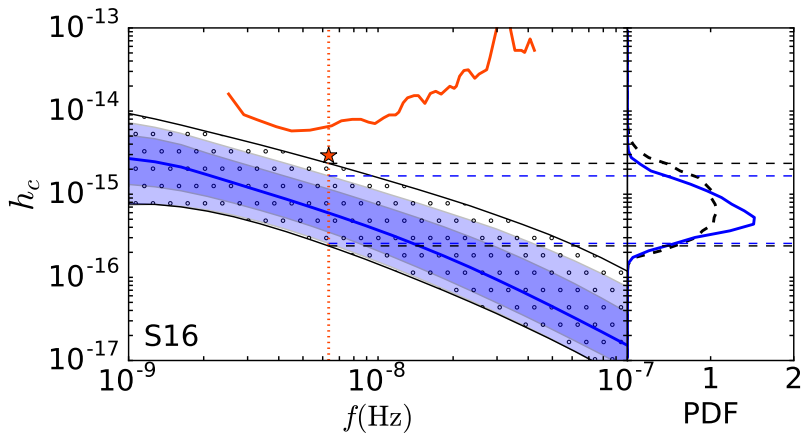
NESTED SAMPLING
cpnest – Veitch & Del Pozzo
github.com/johnveitch/cpnest

Method Overview



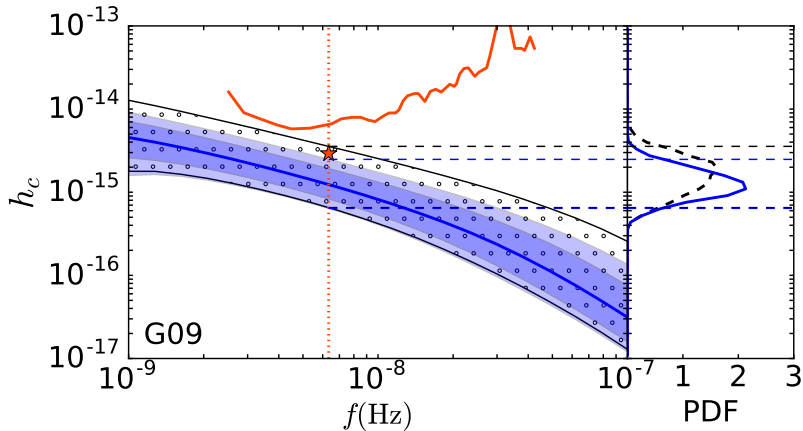
Results

pessimistic



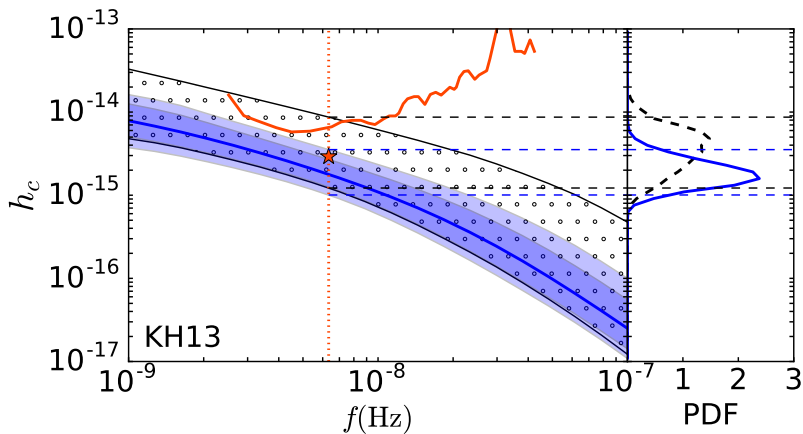
Results

middling



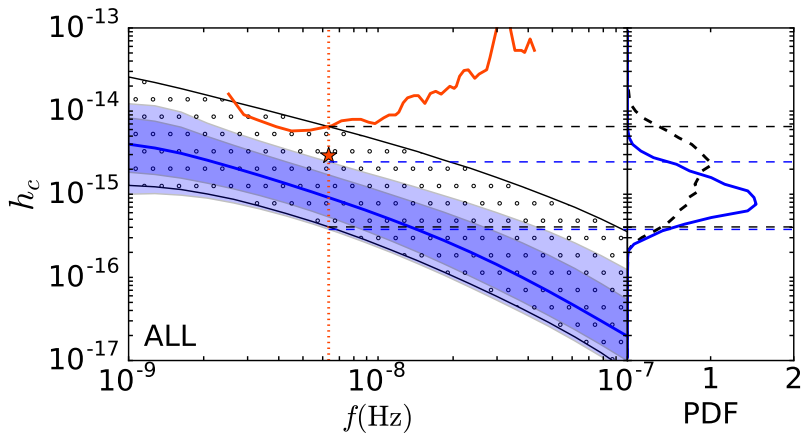
Results

optimistic



Results

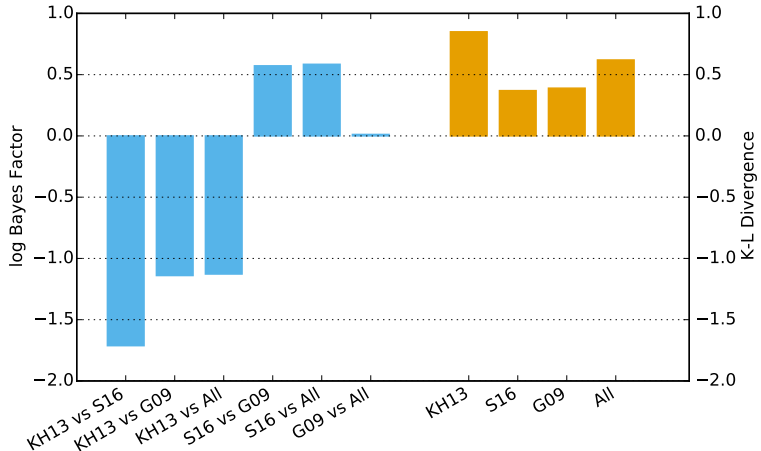
all+



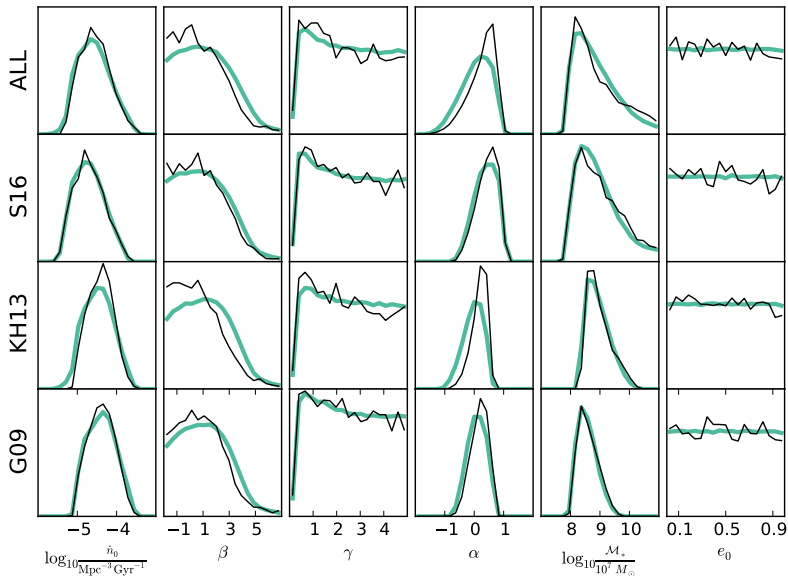
Summary

- Upper limits are eating into predicted values
- But **models are still consistent with observation**
- Little constraint on the model parameters – eccentricity not essential
- No need to rethink predictions yet
- Order of magnitude improvement in sensitivity would put optimistic predictions in more trouble

Bonus slides! Quantitative Results



Bonus slides! Parameters

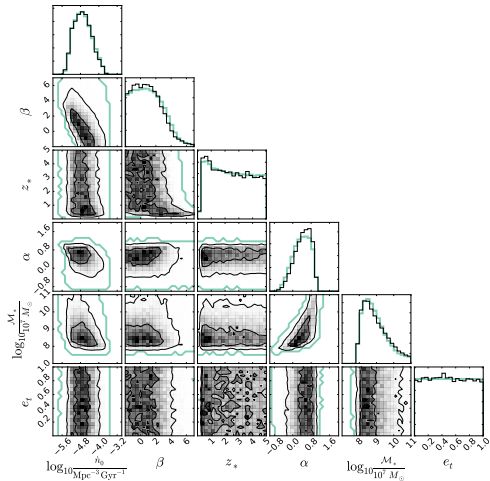


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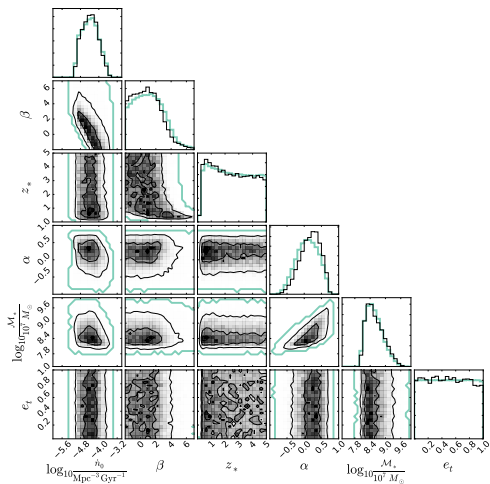
Bonus slides! Parameters

pessimistic



Bonus slides! Parameters

middling

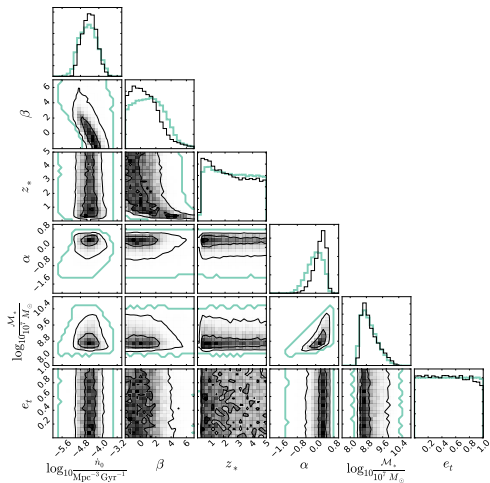


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Bonus slides! Parameters

optimistic



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all+

