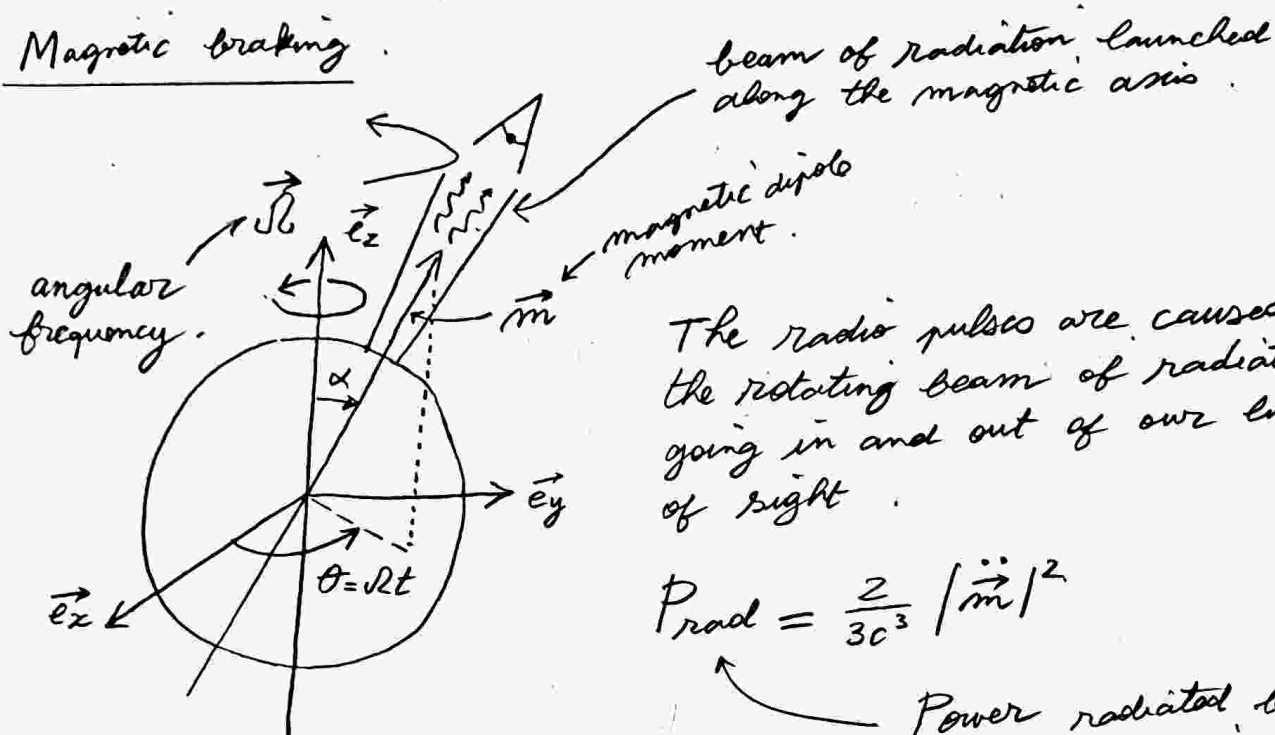


Pulsars

Radio pulses with very stable period $P \approx 1\text{ s}$, increasing at a rate $\dot{P} \approx 10^{-15}$.

Origin from rotating magnetized neutron stars

Magnetic braking



$$P_{\text{rad}} = \frac{2}{3c^3} |\ddot{\vec{m}}|^2$$

Power radiated by an accelerating magnetic dipole.

$$\vec{m} = m \begin{pmatrix} \sin \alpha \cos(\Omega t) \\ \sin \alpha \sin(\Omega t) \\ \cos \alpha \end{pmatrix} \quad \Omega \equiv |\vec{\Omega}|$$

$$\ddot{\vec{m}} = m \begin{pmatrix} -\sin \alpha \Omega^2 \cos(\Omega t) \\ -\sin \alpha \Omega^2 \sin(\Omega t) \\ 0 \end{pmatrix}$$

constant magnitude

$$\text{So, } |\ddot{\vec{m}}|^2 = m^2 \sin^2 \alpha \Omega^4 (\underbrace{\cos^2(\Omega t) + \sin^2(\Omega t)}_1) = m^2 \sin^2 \alpha \Omega^4$$

$$B_p = \frac{2m}{R^3} \quad (\text{for a dipolar field})$$

magnetic field at the neutron star's pole.

neutron star radius

$$P_{\text{rad}} = \frac{2}{3c^3} m^2 \sin^2 \alpha \Omega^4 = \frac{B_p^2 R^6 \sin^2 \alpha \Omega^4}{6c^3}$$

Magnetic dipole radiation results in a loss of rotational energy.

$$\frac{d}{dt} E_{\text{rot}} = -P_{\text{rad}} \quad (\text{energy conservation})$$

rotational energy

$$E_{\text{rot}} = \frac{1}{2} I \Omega^2 \Rightarrow \frac{dE_{\text{rot}}}{dt} = I \dot{\Omega} \Omega$$

moment of inertia.

$$I \dot{\Omega} \Omega = - \frac{B_p^2 R^6 \sin^2 \alpha \Omega^4}{6 c^2}$$

$\frac{2}{5} MR^2$ (for a perfect sphere)

$$S_0 = \boxed{\dot{\Omega} = - \frac{5}{12} \frac{R^4}{M c^3} (B_p \sin \alpha)^2 \Omega^3}$$

Pulsar magnetic braking law.

$$\dot{\Omega} = -K \Omega^m$$

braking index
 $m=3$
(for magnetic braking).

$$K = \frac{5}{12} \frac{R^4}{M c^3} (B_p \sin \alpha)^2$$

$$B_p \sin \alpha = \left(-\frac{12}{5} \frac{M c^3}{R^4} \frac{\dot{\Omega}}{\Omega^3} \right)^{1/2} = \left(\frac{3}{5 \pi^2} \frac{M c^3}{R^4} \frac{P \dot{P}}{P^2} \right)^{1/2}$$

$P = \frac{2\pi}{\Omega}$
 $\frac{\dot{\Omega}}{\Omega^3} = \frac{-2\pi \dot{P}}{P^2} \left(\frac{P}{2\pi} \right)^3 = -\frac{1}{4\pi^2} \frac{P \dot{P}}{P^2}$

$M \approx 1 M_\odot$
 $R \approx 10 \text{ km}$

$$\approx \left(\frac{3 \times 2 \times 10^{33} \times (3 \times 10^{-15})^3}{5 \times 10 \times (10^5)^4} 10^{-15} \right)^{1/2} \approx \left(10^{33+30-15-24} \right)^{1/2} = 10^{12} \text{ G}$$

$$P \dot{P} = 10^{-15} \text{ s}$$

(as observed)

Very strong magnetic field
($B_0 \approx 1 \text{ G}$ for the Sun)

$$\dot{\Omega} = -K \Omega^n \Rightarrow \ddot{\Omega} = -K n \dot{\Omega} \Omega^{n-1} = \frac{\dot{\Omega}^2}{\Omega} n \Rightarrow n = \frac{\ddot{\Omega} \Omega}{\dot{\Omega}^2}$$

$$So: n = - \left(\frac{P^2}{2\pi \dot{P}} \right)^2 \frac{2\pi}{P} \times 2\pi \left(\frac{\ddot{P}}{P^2} - 2 \frac{\dot{P}^2}{P^3} \right) = 2 - \frac{P \ddot{P}}{\dot{P}^2}$$

$$\Omega = \frac{2\pi}{P} \Rightarrow \begin{cases} \dot{\Omega} = -\frac{2\pi \dot{P}}{P^2} \\ \ddot{\Omega} = -2\pi \left(\frac{\ddot{P} P^2 - \dot{P} \times 2\dot{P}P}{P^4} \right) \\ = -2\pi \left(\frac{\ddot{P}}{P^2} - 2 \frac{\dot{P}^2}{P^3} \right) \end{cases}$$

Possible to obtain the braking index n if we can measure $\ddot{P}$.
Typically, observations find $n \approx 2.5 - 3$ (close but lower than expected for magnetic braking).

Can be used to determine the age of a pulsar.

$$\tau = \int_0^t dt = \int_{\Omega_0}^{\Omega} \frac{-d\Omega}{K \Omega^n} = \frac{1}{K} \left[\frac{\Omega^{-n+1}}{n-1} \right]_{\Omega_0}^{\Omega}$$

$\frac{d\Omega}{dt} = -K \Omega^n \equiv \Omega(0)$

$$= \frac{\Omega^{-n+1}}{K(n-1)} \left(1 - \underbrace{\left(\frac{\Omega_0}{\Omega} \right)^{-n+1}}_{\substack{\uparrow \\ 1}} \right)$$

assuming $n > 1$
and $\Omega \ll \Omega_0$

$$So: \tau \approx \frac{\Omega}{\dot{\Omega}(n-1)} = \frac{P}{\dot{P}(n-1)}$$

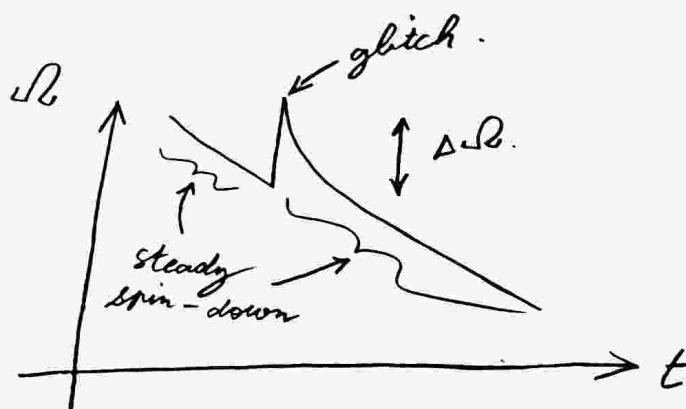
For the pulsar in the Crab nebula $\tau = 1600$ yr.

$\begin{cases} \cdot P = 3.31 \times 10^{-2} \text{ s} \\ \cdot \dot{P} = 4.22 \times 10^{-13} \\ \cdot n = 2.515 \end{cases}$

This age is consistent with an associated supernova observed in 1054!

Pulsar glitches

Discontinuous changes in the pulsar's rotational frequency with $\frac{\Delta\Omega}{\Omega} \approx 10^{-6}$.



Due to a change in the moment of inertia.

$$\frac{\Delta\Omega}{\Omega} \approx \frac{-\Delta I}{I} \approx \frac{-\Delta R}{R} \Rightarrow \text{Requires } |\Delta R| \approx 10^{-6} R \approx 1 \text{ cm!}$$

$\Omega^2 \propto 1/I$
↑
constant rotational energy over the glitch timescale.

$I \propto R^2$
↑
constant mass

Neutron star's radius $R \approx 10 \text{ km}$

The decrease in radius could be due to a reduced centrifugal force, which causes the neutron star's crust to contract to a new equilibrium. During this process, the crust has to crack, producing a "starquake". The rotation of the crust increases until it is slowed down due to friction with the slower neutron star's superfluid interior.