

Supernovae

Flare of luminosity $L_{SN} \approx 10^{43} \text{ erg/s}$ with duration $\Delta t \approx 100 \text{ days}$.
Origin from the collapse and resulting explosion of stars.

Different categories:

- Type I = no hydrogen due to absence of outer envelope.
(includes Type Ia supernovae coming from the explosion of white dwarfs).
- Type II = hydrogen present (comes from the core collapse of massive stars).

Focusing on core-collapse supernovae but similar process for type Ia.

Energetics

Core collapses and bounces back, injecting energy into the envelope that gets ejected.

$$t_{\text{collapse}} \approx \frac{R_{\text{core}}}{v_{\text{core}}} \approx \left(\frac{R_{\text{core}}^3}{G M_{\text{core}}} \right)^{1/2} \approx 10^{-3} \left(\frac{R_0^3}{G M_0} \right)^{1/2} \approx 1 \text{ s}!$$

(timescale of collapse)

free-fall velocity
 $v_{\text{core}} \approx \sqrt{\frac{G M_{\text{core}}}{R_{\text{core}}}}$

$\begin{cases} R_{\text{core}} \approx R_{\text{WD}} = 10^{-2} R_0 \\ M_{\text{core}} \approx M_{\text{ch}} \approx 1 M_\odot \end{cases}$ (for the collapse to happen)

1000 s (dynamical time of the Sun)

$$E_{SN} \approx \frac{G M_{\text{core}}^2}{R_{\text{core}}} \approx \frac{7 \times 10^{-8} \times (2 \times 10^{33})^2}{10^{-2} \times 7 \times 10^{10}} \approx 10^{51} \text{ erg}$$

for the core to be ejected

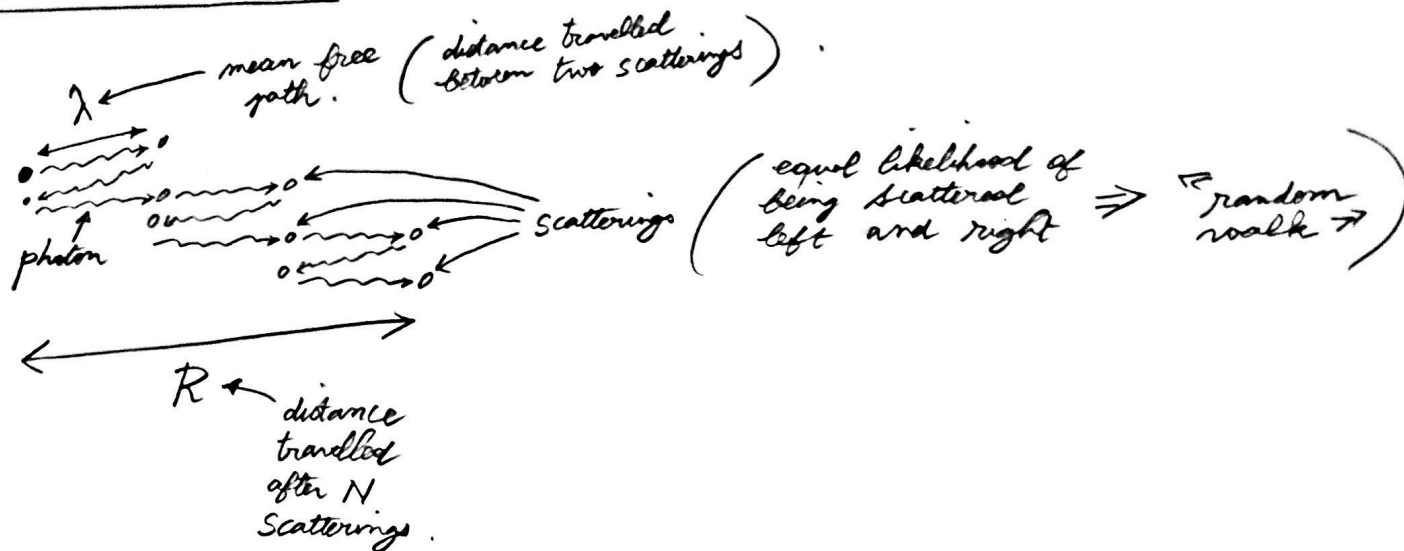
$$E_{\text{ejecta properties}} = \begin{cases} v = \left(\frac{2 E_{SN}}{M} \right)^{1/2} \approx \left(\frac{2 \times 10^{51}}{2 \times 10^{33}} \right)^{1/2} \approx 10^4 \text{ km/s} \\ R = R_0 + vt \end{cases}$$

$M \approx 1 M_\odot$ (ejecta mass).

velocity

Stays constant because no further acceleration after the ejecta has been launched.

Radiative diffusion



$$R = \sqrt{N} \lambda \Rightarrow N = \left(\frac{R}{\lambda}\right)^2$$

for a random walk.

total path length travelled by photons

$$t_{\text{diff}} \equiv \frac{N\lambda}{c} = \left(\frac{R}{\lambda}\right)^2 \frac{\lambda}{c} = \frac{R}{\lambda} \frac{R}{c}$$

diffusion timescale (needed for photons to get to a distance R).

photons travel at the speed of light between scatterings

for an optically thick medium

$$\text{optical depth } \tau \equiv \frac{R}{\lambda} = R \kappa_F \gg 1$$

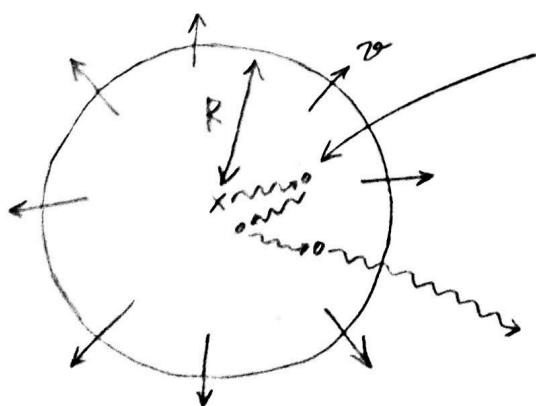
$\lambda = 1/\kappa_F$ opacity ($\text{in cm}^2 \text{g}^{-1}$)

So: $t_{\text{diff}} = \frac{\tau R}{c}$

Diffusion timescale.

(photons effectively move at a speed $v_{\text{diff}} \equiv \frac{R}{t_{\text{diff}}} = \frac{c}{\tau} \ll c$)

Lightwave



Luminosity due to photons that have diffused through the entire ejecta.

$$t_{\text{diff}} = \frac{\tau R}{c} = \frac{\kappa \rho R^2}{c} \approx \frac{3 \kappa M}{4 \pi c R} \approx \frac{3 \kappa M}{4 \pi c v t}$$

$$\rho \approx \frac{M}{\frac{4 \pi}{3} R^3} \quad R = R_0 + v t \approx v t \quad t \gg R_0/v$$

(drops with time as the ejecta get so dense).

Photons can emerge when $t_{\text{diff}} \approx t_{\text{dyn}} \equiv \frac{R}{v}$, as they can catch up with the expanding ejecta.

$$t_{\text{diff}} \approx \frac{R}{v} \approx t \Rightarrow t = \left(\frac{3 \kappa M}{4 \pi c v} \right)^{1/2} \equiv t_{\text{lc}}$$

timescale at which we expect the lightcurve to start dropping.

$$t_{\text{lc}} \approx \left(\frac{10^{33}}{10^{10} \times 10^9} \right)^{1/2} = 10^7 \text{ s} \approx 100 \text{ days} \quad (\text{consistent with supernova timescale})$$

- $M = 1 M_\odot$
- $v \approx 10^4 \text{ km/s}$
- $\kappa \approx 1 \text{ cm}^2/\text{g}$ (for scattering by free electrons)

Now computing the luminosity

$$L = 4 \pi R^2 \underbrace{\rho \frac{v_{\text{diff}}}{3}}_{\text{flux of energy through a surface (the "1/3" accounts for the different directions of diffusion in 3D)}} = \frac{4 \pi R^2}{3 V} \frac{E C}{\tau} = \frac{4 \pi R}{3 \kappa M} E C = \frac{4 \pi v c}{3 \kappa M} E t'$$

radiation energy density $\rho = E/V$

radiation energy

flux of energy through a surface (the "1/3" accounts for the different directions of diffusion in 3D)

$\tau = \kappa R$

$\rho V = M$

$R = R_0 + v t = v t'$

$t' \equiv t + t_0$ where $t_0 \equiv R_0/v$

$$\text{So: } L = \frac{E t'}{t_{\text{lc}}^2}$$

We need to evaluate how E evolves with time using the first principle of thermodynamics.

$$\frac{dE}{dt} = - \underbrace{P \frac{dV}{dt}}_{\text{radiation pressure}} - \underbrace{L}_{\text{radiative energy loss due to photons leaving the ejecta}}$$

« adiabatic energy loss due to expanding volume »

radiation pressure
 $P = \frac{E}{3V}$

$$P \frac{dV}{dt} = \frac{E}{3V} \frac{3V}{t'} = \frac{E}{t'}$$

$$\left\{ \begin{array}{l} P = E/3V \\ \frac{dV}{dt} = \frac{dV}{dt'} = \frac{3V}{t'} \end{array} \right.$$

$$t = t' - t_0$$

$$V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} v^3 t'^3$$

Setting $L=0$:

$$\frac{dE}{dt'} = -\frac{E}{t'} \Rightarrow \frac{dE}{E} = -\frac{dt'}{t'} \Rightarrow \ln E - \ln t' = A \quad \begin{array}{l} \text{constant} \\ \downarrow \end{array}$$

$$\frac{E'}{t'} = e^A \equiv B$$

Initial conditions: $E = E_0$ at $t' = t + t_0 = t_0 \Rightarrow B = \frac{E_0}{t_0}$

$\uparrow$
 $t=0$

So: $E = \frac{E_0 t_0}{t'}$ (decrease of energy due to adiabatic losses only)

Back to $L \neq 0$:

Defining $\Psi(t') = Et'$

$$L = \frac{Et'}{t_0^2} = \frac{\Psi}{t_0}$$

$$\frac{d\Psi}{dt'} = \frac{dE}{dt'} t' + E = -E + E - L t' = -\frac{\Psi t'}{t_0^2}$$

$$\frac{dE}{dt'} = -\frac{E}{t'} - L$$

(first principle of thermodynamics)
 with $P \frac{dV}{dt} = E/t'$

$$\frac{d\psi}{\psi} = -\frac{t' dt'}{t_{ec}^2}$$

$$\ln \psi(t') - \ln \psi(t_0) = -\frac{1}{2t_{ec}^2} (\underbrace{t'^2 - t_0^2}_{t^2 + 2tt_0 \approx t^2}) \quad t' = t + t_0$$

$E \cdot t_0$

$$t^2 + 2tt_0 \approx t^2$$

↑
assuming
here $t \gg t_0$
for simplicity.

$$\psi = E \cdot t_0 \cdot \exp\left(-\frac{1}{2}\left(\frac{t}{t_{ec}}\right)^2\right)$$

$$\text{So: } L = \frac{\psi}{t_{ec}^2} = \left(\frac{E \cdot t_0}{t_{ec}^2}\right) \exp\left(-\frac{1}{2}\left(\frac{t}{t_{ec}}\right)^2\right)$$

As expected, t_{ec} is the timescale at which the luminosity starts dropping.

$$L_{\text{peak}} = \frac{E_0 \cdot \underbrace{(t/t_{ec})}_{\text{energy reduction due to adiabatic losses}}}{t_{ec} \leftarrow \text{timescale over which energy is radiated.}}$$

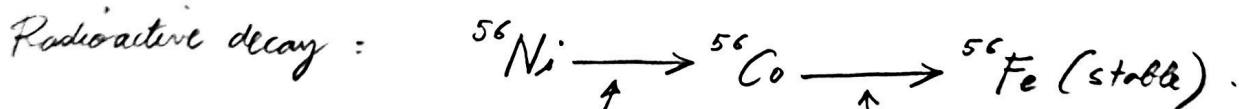
$$L_{\text{peak}} \approx \frac{10^{51} \times 100}{(10^7)^2} = 10^{53-14} = 10^{39} \text{ erg/s} \ll L_{\text{SN}} \approx 10^{43} \text{ erg/s.}$$

$$t_0 = \frac{R_0}{v} \approx \frac{7 \times 10^{10}}{10^9} \approx 100 \text{ s.}$$

$R_0 = 1 R_\odot$

Too low to explain supernovae.

Another source of heating is needed to boost the luminosity.



half-life $\tau_{1/2} \approx 6 \text{ days}$
energy per decay $E_d \approx 2 \text{ MeV}$

$\tau_{1/2} \approx 80 \text{ days}$
 $E_d \approx 1 \text{ MeV}$

$$1 \text{ eV} \approx 10^{12} \text{ erg}$$

$$\text{Heating rate: } \dot{Q} \approx N_d \frac{E_d}{\tau_{1/2}} \approx \frac{10^{55} \times 10^6 \times 10^{-12}}{10^6} = 10^{43} \text{ erg/s.}$$

number of decays: $N_d = \frac{M_{\text{Ni}}}{56 \times m_p} \approx \frac{10^{33}}{100 \times 10^{-24}} = 10^{55}$

mass of a ^{56}Ni nucleus

$M_{\text{Ni}} \approx M_\odot$

$\dot{Q} \approx L_{SN} \Rightarrow$ Radioactive decay is enough to boost the luminosity.

The first principle of thermodynamics becomes:

$$\frac{dE}{dt} = -P \frac{dV}{dt} - L + \dot{Q}(t) \leftarrow \text{energy gain from radioactive decay.}$$

$$\frac{dL}{dt} = \frac{1}{t_{lc}^2} \frac{d}{dt} (Et') = \frac{1}{t_{lc}^2} \left(t' \frac{dE}{dt'} + E \right) = \frac{t'}{t_{lc}^2} (\dot{Q} - L)$$

$$t' \left(\underbrace{\frac{dE}{dt'}}_{\substack{= \frac{E}{t'} \\ = \frac{P dV}{dt}}} \right) = t' (\dot{Q} - L)$$

At $t = t_{peak}$, $\frac{dL}{dt} = 0 \Rightarrow \boxed{L(t_{peak}) = \dot{Q}(t_{peak})}$

time at which the lightcurve peaks

Arnett's law
(The luminosity equals the heating rate at the time of peak.)

Full solution (can be obtained numerically) =

