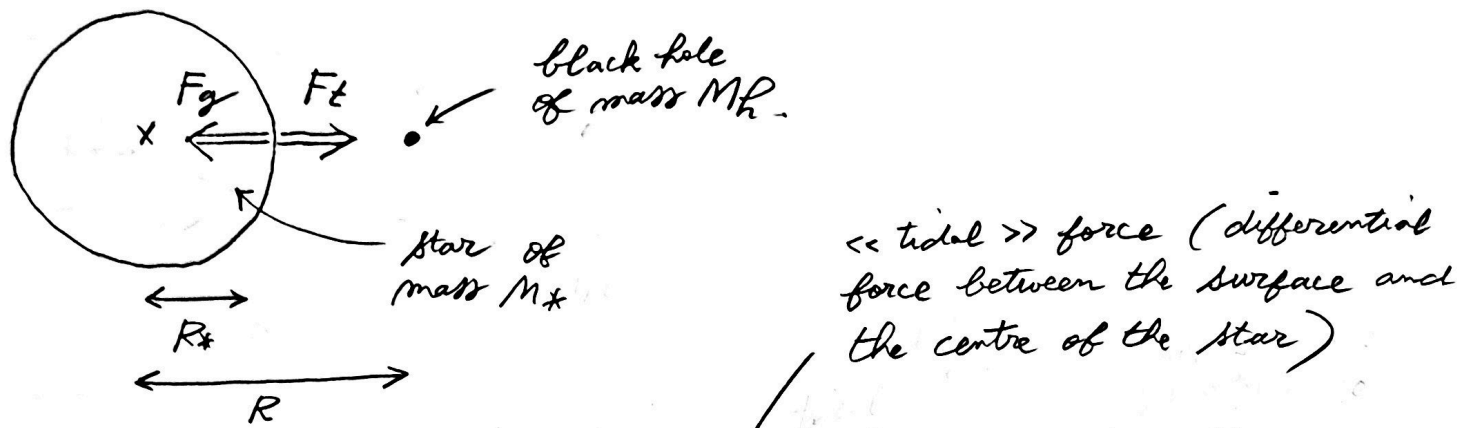


Tidal disruption events

Flare resulting from the disruption of a star by a supermassive black hole. Mostly observed in the X-ray and optical bands with luminosity $L \approx 10^{43} - 10^{44} \text{ erg/s}$.

Tidal radius



The star gets disrupted if : $F_t > F_g$ ← star's self-gravity force.

$$F_g \approx \frac{GM_*}{R_*^2}$$

$$F_t \approx \frac{GM_b}{(R-R_*)^2} - \frac{GM_b}{R^2} \approx \frac{GM_b}{R^3} R_*$$

↑
order of magnitude.

$$\frac{GM_b}{R^2} \left(1 - \frac{R_*}{R}\right)^{-3}$$

SS ← $R_* \ll R$

$$1 + \frac{2R_*}{R}$$

$$\frac{GM_b}{R^2} R_* > \frac{GM_*}{R_*^2}$$

So: $R < R_* \left(\frac{M_b}{M_*}\right)^{1/3} \equiv R_t$

tidal radius (inside which the star is disrupted)

Numerically: $R_t \approx 100 R_\odot \approx 0.7 \text{ AU}$.

$$\begin{cases} \cdot M_h = 10^5 M_\odot \\ \cdot M_* = 1 M_\odot \\ \cdot R_* = 1 R_\odot \end{cases}$$

$$\begin{cases} \cdot R_\odot \approx 7 \times 10^{10} \text{ cm} \\ \cdot 1 \text{ AU} \approx 10^{13} \text{ cm} \end{cases}$$

↑
astronomical unit

Hills' mass limit

A tidal disruption event is only detectable if the star is disrupted outside the black hole's event horizon.

$$\underbrace{R_t}_{\parallel R_* \left(\frac{M_h}{M_*}\right)^{1/3}} > 2 R_g = \frac{2 G M_h}{c^2} \Rightarrow M_h^{2/3} < \frac{R_*}{M_*^{1/3}} \frac{c^2}{2 G}$$

$$\text{So } M_h < \frac{c^3}{M_*^{1/2}} \left(\frac{R_*}{2 G}\right)^{3/2} = \boxed{M_* \left(\frac{c^2 R_*}{2 G M_*}\right)^{3/2} \equiv M_H}$$

Hills' mass limit for the black hole (below which the disruption is detectable).

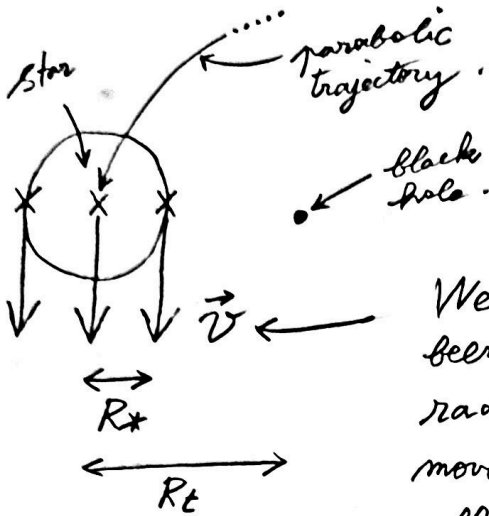
Numerically: $M_H \approx M_\odot \times \left(\frac{7 \times 10^{10} \text{ cm}}{3 \text{ km}}\right)^{3/2} \approx 10^8 M_\odot$

$$\begin{cases} \cdot M_* = 1 M_\odot \\ \cdot R_* = 1 R_\odot \end{cases}$$

$M_h \lesssim 10^8 M_\odot$ corresponds to the low-mass end of supermassive black holes.

Process of disruption:

The star comes from a distance from the black hole much larger than the tidal radius, such that its trajectory can be approximated as parabolic with specific orbital energy $E_* = 0$.



We assume that the star has not yet been deformed when reaching the tidal radius, such that all its fluid elements move at the same velocity $\vec{v}$. This is called the "frozen-in" approximation.

specific orbital energy for the star's center of mass

$$E(R_t) = \frac{1}{2} \vec{v}^2 - \frac{GM R}{R_t} = E_* = 0$$

parabolic trajectory

$$E(R_t - R_*) = \frac{1}{2} \vec{v}^2 - \frac{GM R}{R_t - R_*} = \frac{GM R}{R_t} \left(1 - \frac{R_*}{R_t}\right)^{-1}$$

specific orbital energy for the fluid element closest to the black hole.

$$\approx \frac{GM R}{R_t} + \frac{GM R}{R_t^2} R_*$$

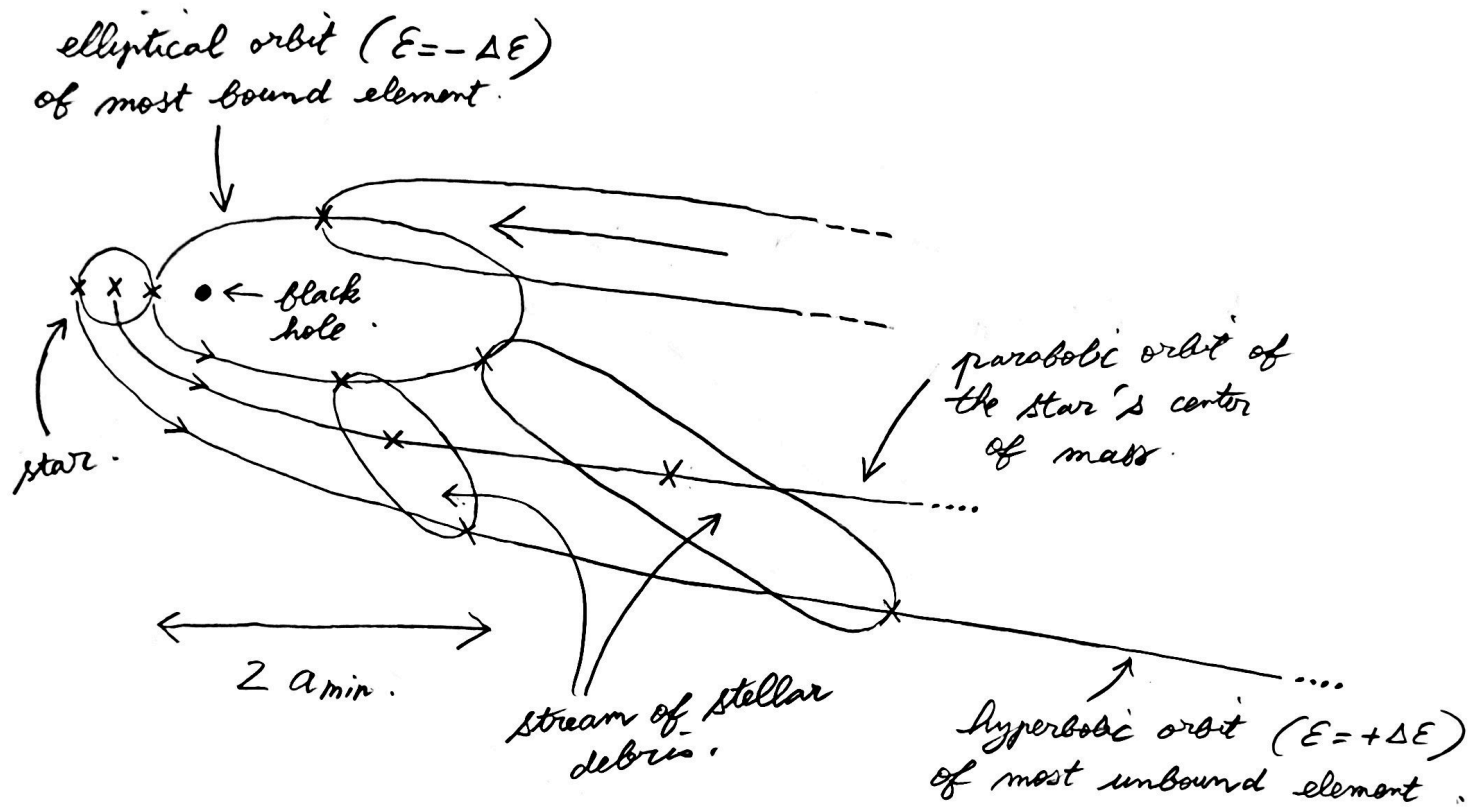
$\frac{R_*}{R_t} \ll 1$

$$E(R_t - R_*) \approx \frac{1}{2} \vec{v}^2 - \frac{GM R}{R_t} - \frac{GM R}{R_t^2} R_* = - \frac{GM R}{R_t^2} R_*$$

for the element furthest from the black hole

Similarly: $E(R_t + R_*) \approx + \frac{GM R}{R_t^2} R_*$

So, the gas gets imparted a spread in specific orbital energy given by: $\Delta E = \frac{GM_R}{R_t^2} R_*$. The elements located at $R < R_t$ (the half of the star closest to the black hole) become bound with energies $E \in [-\Delta E, 0] < 0$, while the elements at $R > R_t$ (the other half) become unbound with energies $E \in [0, \Delta E] > 0$.



The star gets stretched into a stream of gas debris, half of which falls back towards the black hole. This bound gas returns to pericenter in a range of times $t \in [t_{min}, +\infty[$ after the disruption, given by their orbital periods.

$$t = 2\pi \left(\frac{a^3}{GM_R} \right)^{1/2} = 2\pi GM_R (-2E)^{-3/2}$$

↑ orbital period of bound debris

↑ Kepler's third law

↑ $a = -\frac{GM_R}{2E}$

↑ semi-major axis

↑ orbital energy

$$t_{\min} = 2\pi G M_L (2\Delta E)^{-3/2} = 2\pi G M_* \left(\frac{2 G M_*}{R_*} \right)^{-3/2} \left(\frac{M_L}{M_*} \right)^{1/2}$$

$\uparrow$
 $E = -\Delta E$
 for the most
 bound debris

$$\Delta E = \frac{G M_L}{R_t^2} R_* = \frac{G M_*}{R_*} \left(\frac{M_L}{M_*} \right)^{1/3}$$

$\uparrow$
 $R_t = R_* \left(\frac{M_L}{M_*} \right)^{1/3}$

So:

$$t_{\min} = \frac{\pi}{\sqrt{2}} \left(\frac{R_*^3}{G M_*} \right)^{1/2} \left(\frac{M_L}{M_*} \right)^{1/2}$$

Orbital period of the most bound debris, from which the stream starts coming back to pericenter.

Numerically = $t_{\min} \approx 41$ days

$\uparrow$
 $\begin{cases} \cdot M_* = 1 M_\odot \\ \cdot R_* = 1 R_\odot \\ \cdot M_L = 10^6 M_\odot \end{cases}$

Fallback rate

We now compute the "fallback rate" at which the stream returns to pericenter.

$$\dot{M}_{\text{fb}} = \frac{dM}{dt} = \left(\frac{dM}{dE} \right) \frac{dE}{dt} \propto t^{-5/3}$$

$\uparrow$
 fallback rate

$\uparrow$
 mass of returning stream per bin of orbital energy

$\nwarrow$ $\frac{dE}{dt} \propto t^{-5/3}$ (because $E \propto t^{-2/3}$)
 $\uparrow$
 assuming that dM/dE is constant

So:

$$\dot{M}_{\text{fb}} \approx \dot{M}_p \left(\frac{t}{t_{\min}} \right)^{-5/3}$$

Mass fallback rate of the debris stream.

peak fallback rate.

$$\int_{t_{\min}}^{+\infty} \dot{M}_{fb} dt = \dot{M}_p \int_{t_{\min}}^{+\infty} \left(\frac{t}{t_{\min}}\right)^{-5/3} dt = \frac{3}{2} \dot{M}_p t_{\min}$$

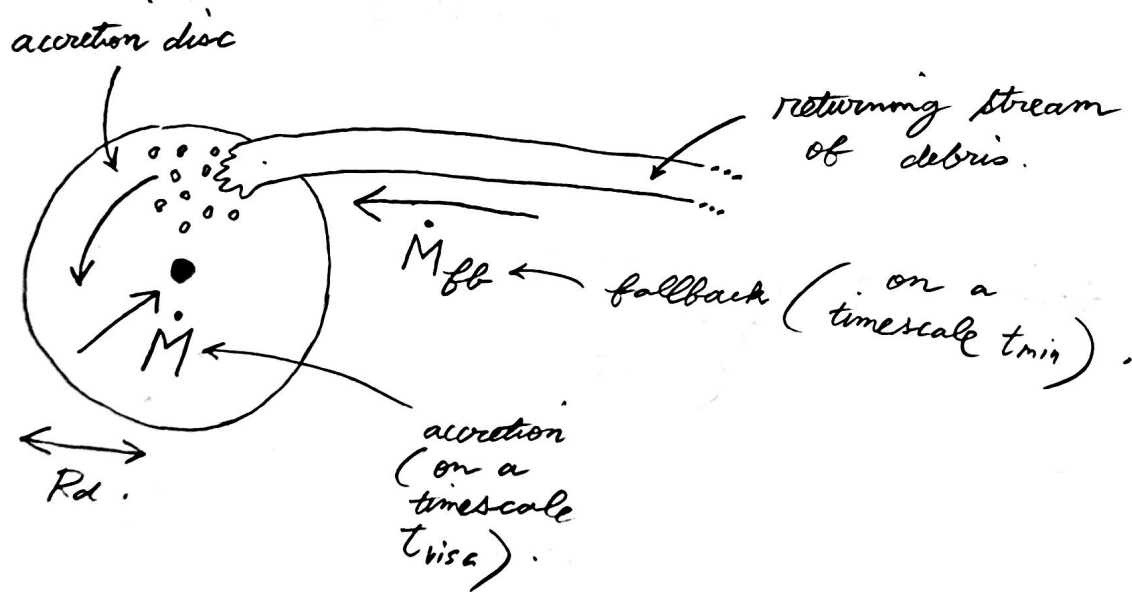
$$\frac{M_*}{2} \quad \parallel \quad \left[-\frac{3}{2} \left(\frac{t}{t_{\min}}\right)^{-2/3} t_{\min} \right]_{t_{\min}}^{+\infty} = \frac{3}{2} t_{\min}$$

only half of the star's mass is bound.

$$\text{So: } \dot{M}_p = \frac{M_*}{3 t_{\min}} \approx 3 M_{\odot} \text{ yr}^{-1}$$

Accretion luminosity

We assume that an accretion disc forms that is progressively joined by the returning stream.



$$t_{\text{visc}} = \frac{R_d^2}{\nu} = \frac{1}{\alpha} \left(\frac{H}{R_d}\right)^{-2} \left(\frac{R_d^3}{G M_R}\right)^{1/2}$$

viscous timescale of the disc.

$$\nu = \alpha C_s H \leftarrow \text{disc width}$$

viscosity ($\alpha \approx 0.1$, see lecture on accretion)

sound speed

$$C_s \approx \frac{H}{R_d} v_K$$

see lecture on accretion

$$v_K = \left(\frac{G M_R}{R_d}\right)^{1/2}$$

$$t_{\text{visc}} \approx \left(\frac{1}{\alpha} \right)^{10} \left(\frac{R_t^3}{GM_R} \right)^{1/2} \approx 10 \left(\frac{R_*^3}{GM_*} \right)^{1/2} \approx 10^4 \text{ s} \approx 0.1 \text{ day} \ll t_{\text{min}}$$

$\left\{ \begin{array}{l} \cdot H/R_d \approx 1 \text{ (because we will find } L \approx L_{\text{Edd}}) \\ \cdot R_d \approx R_t \end{array} \right.$

angular momentum conservation.

$$\left(\begin{array}{l} \uparrow \\ \text{star's specific} \\ \text{angular momentum} \end{array} \right. \ell_* = \ell_d \xrightarrow{\quad} R_d = 2R_t \left. \begin{array}{l} \uparrow \\ \text{same for disc} \end{array} \right)$$

$$\left\{ \begin{array}{l} \cdot \ell_*^2 = 2GM_R R_t \text{ (parabolic orbit)} \\ \cdot \ell_d^2 = GM_R R_d \text{ (circular orbit)} \end{array} \right.$$

Because $t_{\text{visc}} \ll t_{\text{min}}$, the mass from the returning stream joining the disc promptly flows towards the black hole with an accretion rate $\dot{M} \approx \dot{M}_{\text{fb}}$.

radiative efficiency $\rightarrow$ peak luminosity

$$L = \eta \dot{M} c^2 \xrightarrow{\dot{M} \approx \dot{M}_{\text{fb}}} L \approx L_p \left(\frac{t}{t_{\text{min}}} \right)^{-5/3} \propto t^{-5/3}$$

$\uparrow$ luminosity from the accretion disc.

$$L \approx L_p \left(\frac{t}{t_{\text{min}}} \right)^{-5/3} \propto t^{-5/3}$$

Accretion luminosity from a tidal disruption event with a characteristic decay law of $t^{-5/3}$.

$$L_p \approx \eta \dot{M}_p c^2 \approx 10^{46} \text{ erg/s} > L_{\text{Edd}} \approx 10^{44} \text{ erg/s}$$

$\uparrow$ peak luminosity

$$\left\{ \begin{array}{l} \cdot M_R = 10^6 M_\odot \\ \cdot M_* = 1 M_\odot \\ \cdot R_* = 1 R_\odot \\ \cdot \eta = 0.1 \end{array} \right.$$

At early times, the luminosity is limited to $L \approx L_{\text{Edd}}$.

The fact that $L_p > L_{\text{Edd}}$ implies that the radiation pressure force launches the gas into outflows.

While the accretion disc emits X-rays, some of these photons may be absorbed by the surrounding gas outflows before being re-emitted at lower energies. This "reprocessing" mechanism is one of the possible origin for the optical emission from tidal disruption events.