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Any Calculator
Open Note

UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF BSc & MSci WITH HONOURS

THIRD YEAR EXAMINATION

03 00716

Observational Cosmology

Total time allowed: 1 hour 30 minutes

SUMMER EXAMINATIONS 2013

Answer Section 1 and any one question from Section 2.

Section 1 counts for 40% of the marks for the examination. Full marks may be obtained by correctly answering four questions. You may attempt as many questions as you wish, but any marks in excess of 40% will be disregarded.

Section 2 consists of two questions, each carrying 30% of the marks. If you answer more than one question from this Section, only the first will be marked.

The approximate allocation of marks to each part of a question is shown in brackets [ ]. A further 30% of the course credit derives from coursework assignments already submitted.

Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

Students may use their course notes in the examination, but textbooks are not permitted

A table of physical constants and units that may be required will be found at the end of this question paper.

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SECTION 1

Full marks may be obtained by correctly answering four questions. You may attempt as many questions as you wish, but any marks in excess of 40% will be disregarded.

1. Is it possible for a closed universe to evolve to become an open universe? Give a reason for your answer. **[10]**

2. Consider a Λ-dominated universe. Calculate the value of the deceleration parameter at a time when the scale factor is $a = a_0$, and at a later time when such a universe has $a = 2a_0$. **[10]**

3. Consider an open universe. Determine which component – matter, radiation, curvature or cosmological constant – dominates the late time dynamics of such a universe. **[10]**

4. Estimate the redshift at which the evolution of our Universe becomes dark-energy dominated. **[10]**

5. Suppose we live in a closed universe ($k > 0$) that will re-collapse some time in the future. What will the temperature of the radiation field be when the universe has gone through its maximum size and then shrunk back to *half* its present size? **[10]**

6. Consider the dipole anisotropy present in the Cosmic-Microwave-Background (CMB) maps. This anisotropy component produces a maximum variation of temperature, with respect to its average value, of $+0.0034$ K in a direction that we identify with the unit vector $\hat{\Omega}$. Write down the angular dependence of the temperature variation with respect to $\hat{\Omega}$. Calculate the velocity of the solar system with respect to the CMB rest-frame and the value of the temperature in the directions opposite and perpendicular to $\hat{\Omega}$. **[10]**

SECTION 2

You should attempt one question from this Section. If you answer more than one question, only the first one will be marked.

7. Type Ia supernovae are considered standard candles and as such they provide one of the best ways to obtain precise measurements of the cosmological parameters. In reality it is known that they exhibit *intrinsic* luminosity differences.

   (a) If the luminosity of supernovae has an intrinsic rms scatter of 10%, evaluate the fractional error of the inferred luminosity distance. **[12]**

   (b) For a low redshift supernova at $z = 0.026$, determine the error in the luminosity distance arising from this spread in intrinsic luminosity. **[8]**

   (c) The error in supernovae distance measurements can be reduced by averaging over a large number of supernovae. Calculate how many supernovae are required to achieve a fractional error of 1%. **[5]**

   (d) Consider now a universe with $k > 0$. By deriving an expression of the luminosity distance as a function of the proper distance, calculate whether the luminosity distance reduces to the proper distance for nearby (with respect to cosmological scales) objects. **[5]**

8. Consider a flat, matter-dominated universe.

   (a) Derive an expression for the angle subtended by an object of physical size L as a function of redshift. **[15]**

   (b) Calculate the redshift at which the object appears the smallest. **[10]**

   (c) Find the asymptotic behaviour of the angle above as a function of redshift in the limit of small and large z. **[5]**

Physical Constants and Units

(For all years)

Acceleration due to gravity	g	$9.81\ \mathrm{ms^{-2}}$
Gravitational constant	G	$6.673 \times 10^{-11}\ \mathrm{Nm^{-2}kg^{-2}}$
Avogadro constant	N_A	$6.022 \times 10^{23}\ \mathrm{mol^{-1}}$
	Note: 1 mole = 1 *gram* molecular-weight	
Ice point	T_ice	273.15 K
Gas Constant	R	$8.314\ \mathrm{JK^{-1}mol^{-1}}$
Boltzmann constant	k, k_B	$1.381 \times 10^{-23}\ \mathrm{JK^{-1}} = 0.862 \times 10^{-4}\ \mathrm{eV\ K^{-1}}$
Stefan constant	σ	$5.670 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$
Rydberg constant	R_∞	$1.097 \times 10^{7}\ \mathrm{m^{-1}}$
	$R_\infty hc$	$13.61\ \mathrm{eV}$
Planck constant	h	$6.626 \times 10^{-34}\ \mathrm{Js} = 4.136 \times 10^{-15}\ \mathrm{eV\ s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34}\ \mathrm{Js} = 6.582 \times 10^{-16}\ \mathrm{eV\ s}$
Speed of light *in vacuo*	c	$2.998 \times 10^{8}\ \mathrm{ms^{-1}}$
	$\hbar c$	$197.3\ \mathrm{MeV\ fm}$
Charge of proton	e	$1.602 \times 10^{-19}\ \mathrm{C}$
Mass of electron	m_e	$9.109 \times 10^{-31}\ \mathrm{kg}$
Rest energy of electron		$0.511\ \mathrm{MeV}$
Mass of proton	m_p	$1.673 \times 10^{-27}\ \mathrm{kg}$
Rest energy of proton		$938.3\ \mathrm{MeV}$
One atomic mass unit	u	$1.660 \times 10^{-27}\ \mathrm{kg}$
Atomic mass unit energy equivalent		$931.5\ \mathrm{MeV}$
Electric constant	ε_0	$8.854 \times 10^{-12}\ \mathrm{Fm^{-1}}$
Magnetic constant	μ_0	$4\pi \times 10^{-7}\ \mathrm{Hm^{-1}}$
Bohr magneton	μ_B	$9.274 \times 10^{-24}\ \mathrm{Am^2}\ (\mathrm{JT^{-1}})$
Nuclear magneton	μ_N	$5.051 \times 10^{-27}\ \mathrm{Am^2}\ (\mathrm{JT^{-1}})$
Fine structure constant	$\alpha = e^2/4\pi\varepsilon_0\hbar c$	$7.297 \times 10^{-3} = 1/137.0$
Compton wavelength of electron	$\lambda_\mathrm{C} = h/mc$	$2.426 \times 10^{-12}\ \mathrm{m}$
Bohr radius	a_0	$5.292 \times 10^{-11}\ \mathrm{m}$
angstrom	Å	$10^{-10}\ \mathrm{m}$
torr (mmHg, $0°\mathrm{C}$)	torr	$133.3\ \mathrm{Pa}\ (\mathrm{Nm^{-2}})$
barn	b	$10^{-28}\ \mathrm{m^2}$