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UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF BSc & MSci WITH HONOURS

THIRD YEAR EXAMINATION

03 00716

Observational Cosmology

Total time allowed: 1 hour 30 minutes

SUMMER EXAMINATIONS 2014

Answer Section 1 and any one question from Section 2.

Section 1 counts for 40% of the marks for the examination. Full marks may be obtained by correctly answering four questions. You may attempt as many questions as you wish, but any marks in excess of 40% will be disregarded.

Section 2 consists of two questions, each carrying 30% of the marks. If you answer more than one question from this Section, credit will only be given for the best answer.

The approximate allocation of marks to each part of a question is shown in brackets [ ]. A further 30% of the course credit derives from coursework assignments already submitted.

Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

Students may use their course notes in the examination, but textbooks are not permitted

Two tables of physical constants and units that may be required will be found at the end of this question paper.

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SECTION 1

Full marks may be obtained by correctly answering four questions. You may attempt as many questions as you wish, but any marks in excess of 40% will be disregarded.

1. Consider a flat universe made up of matter and a non-classical fluid with equation of state $p = -2\rho$, where ρ is constant and positive. Show whether the acceleration of this universe is constant throughout its expansion and derive the behaviour at late cosmic time. **[10]**

2. The flux of a quasar at $z = 2.3$ is measured to be F. The quasar has an angular diameter distance of 1.7 Gpc. What is the intrinsic luminosity of the quasar? Express the result in terms of F. **[10]**

3. Estimate the temperature T at which the early Universe became transparent to photons by considering the mean photon energy at a given temperature and the ionisation energy of hydrogen. Explain why your result overestimates the actual temperature. **[10]**

4. Suppose it were suggested that the dark matter consists of black holes of mass $10^{-10}M_{\odot}$. Make an order of magnitude estimate of how distant you would expect the nearest such black hole to be, stating your assumptions clearly. **[10]**

5. Discuss why the flatness of our Universe is considered to be a problem for the hot big bang model. **[10]**

6. The mean density of the light from all galaxies is $3.3 \times 10^{8}\, h \mathrm{L}_{\odot}\, \mathrm{Mpc}^{-3}$ and the typical mass-to-light ratio within galaxies is approximately $30\, h M_{\odot}/L_{\odot}$, where h is the Hubble parameter in units of 100 km $\mathrm{s}^{-1}\,\mathrm{Mpc}^{-1}$. Derive the associated mass-density parameter of galaxies at the present epoch and comment on the result. **[10]**

SECTION 2

You should attempt one question from this Section. If you answer more than one question, credit will only be given for the best answer.

7. Consider our Universe.

   (a) Using the fact that the temperature of the CMB today is $T_0 = 2.73$ K, derive the contribution of photons to the density parameter Ω. **[5]**

   (b) What is the luminosity distance of an object at $z = 0.12$? Make suitable approximations in your calculation. **[7]**

   (c) Determine whether the Universe was matter- or radiation-dominated at decoupling (you do not need to derive the redshift at decoupling.) **[5]**

   (d) Estimate the redshift corresponding to the epoch of matter-radiation equality. **[5]**

   (e) Calculate approximately how many years of cosmic evolution have elapsed since the Universe became transparent to photons. **[8]**

8. Consider a universe made up of matter and a cosmological constant contribution, Λ.

   (a) What is the value of Λ, as a function of the matter energy density ρ_m that is needed to allow for a static universe? **[8]**

   (b) Is the curvature of such a universe positive, negative or zero? Justify your answer. **[5]**

   (c) What is the scale factor of such a universe? **[5]**

   (d) What is the total pressure, as a function of ρ_m, that describes such a universe? **[5]**

   (e) Explain what would happen if such a universe were to be perturbed by infinitesimally increasing its scale factor. **[7]**

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Physical Constants and Units

(For all years)

Acceleration due to gravity	g	$9.81\ \mathrm{ms^{-2}}$
Gravitational constant	G	$6.673 \times 10^{-11}\ \mathrm{Nm^{-2}kg^{-2}}$
Avogadro constant	N_A	$6.022 \times 10^{23}\ \mathrm{mol^{-1}}$
	Note: 1 mole = 1 *gram* molecular-weight	
Ice point	T_ice	273.15 K
Gas Constant	R	$8.314\ \mathrm{JK^{-1}mol^{-1}}$
Boltzmann constant	k, k_B	$1.381 \times 10^{-23}\ \mathrm{JK^{-1}} = 0.862 \times 10^{-4}\ \mathrm{eV\ K^{-1}}$
Stefan constant	σ	$5.670 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$
Rydberg constant	R_∞	$1.097 \times 10^{7}\ \mathrm{m^{-1}}$
	$R_\infty hc$	$13.61\ \mathrm{eV}$
Planck constant	h	$6.626 \times 10^{-34}\ \mathrm{Js} = 4.136 \times 10^{-15}\ \mathrm{eV\ s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34}\ \mathrm{Js} = 6.582 \times 10^{-16}\ \mathrm{eV\ s}$
Speed of light *in vacuo*	c	$2.998 \times 10^{8}\ \mathrm{ms^{-1}}$
	$\hbar c$	$197.3\ \mathrm{MeV\ fm}$
Charge of proton	e	$1.602 \times 10^{-19}\ \mathrm{C}$
Mass of electron	m_e	$9.109 \times 10^{-31}\ \mathrm{kg}$
Rest energy of electron		$0.511\ \mathrm{MeV}$
Mass of proton	m_p	$1.673 \times 10^{-27}\ \mathrm{kg}$
Rest energy of proton		$938.3\ \mathrm{MeV}$
One atomic mass unit	u	$1.660 \times 10^{-27}\ \mathrm{kg}$
Atomic mass unit energy equivalent		$931.5\ \mathrm{MeV}$
Electric constant	ε_0	$8.854 \times 10^{-12}\ \mathrm{Fm^{-1}}$
Magnetic constant	μ_0	$4\pi \times 10^{-7}\ \mathrm{Hm^{-1}}$
Bohr magneton	μ_B	$9.274 \times 10^{-24}\ \mathrm{Am^2\ (JT^{-1})}$
Nuclear magneton	μ_N	$5.051 \times 10^{-27}\ \mathrm{Am^2\ (JT^{-1})}$
Fine structure constant	$\alpha = e^2/4\pi\varepsilon_0\hbar c$	$7.297 \times 10^{-3} = 1/137.0$
Compton wavelength of electron	$\lambda_\mathrm{C} = h/mc$	$2.426 \times 10^{-12}\ \mathrm{m}$
Bohr radius	a_0	$5.292 \times 10^{-11}\ \mathrm{m}$
angstrom	Å	$10^{-10}\ \mathrm{m}$
torr (mmHg, $0^\circ\mathrm{C}$)	torr	$133.3\ \mathrm{Pa\ (Nm^{-2})}$
barn	b	$10^{-28}\ \mathrm{m^2}$

Astrophysical Constants and Units

Astronomical Unit	AU	$1.50 \times 10^{11}\,\mathrm{m}$
Parsec	pc	$3.1 \times 10^{16}\,\mathrm{m}$
Tropical year	y	365.242 mean solar days
Solar luminosity	$L_\odot$	$3.84 \times 10^{26}\,\mathrm{W}$
Absolute bolometric magnitude of the Sun	M_{bol}	$+4^{\mathrm{m}}75$
Bolometric correction for the Sun	BC	$-0^{\mathrm{m}}08$
Apparent visual magnitude of the Sun	$m_v(\odot)$	$-26^{\mathrm{m}}74$
Solar constant	f	$1.36 \times 10^{3}\,\mathrm{W\,m^{-2}}$
Solar mass	$M_\odot$	$1.989 \times 10^{30}\,\mathrm{kg}$
Wien constant	b	$2.898 \times 10^{-3}\,\mathrm{m\,K}$
Hubble constant	H_o	$70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$
Solar radius	$R_\odot$	$6.95 \times 10^{8}\,\mathrm{m}$
Distance of the Sun from galactic centre		$8.3\,\mathrm{kpc}$
Earth mass	$M_\oplus$	$5.972 \times 10^{24}\,\mathrm{kg}$
Earth radius	$R_\oplus$	$6371\,\mathrm{km}$