UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF B.Sc. & M.Sci. WITH HONOURS

THIRD-YEAR FINAL EXAMINATION

03 00716

LH OBSERVATIONAL COSMOLOGY

SUMMER EXAMINATIONS 2016

Time Allowed: 1 hour 30 minutes

Answer Section 1 and one question from Section 2.

Section 1 counts for 40% of the marks for the examination.
Full marks for this Section can be obtained by correctly answering ***four*** questions.
You may attempt more questions, but marks in excess of 40% will be disregarded.

Section 2 consists of two questions and carries 30% of the marks.
Answer ***one*** question from this Section. If you answer more than one question, credit will only be given for the best answer.

The approximate allocation of marks to each part of a question is shown in brackets **[ ]**. A further 30% of the course credit derives from coursework assignments already submitted.

All symbols have their usual meanings.

Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

Students may use their course notes in the examination, but textbooks are not permitted.

Two tables of physical constants and units that may be required will be found at the end of this question paper.

SECTION 1

*Full marks for this section can be obtained by correctly answering **four** questions. You may attempt as many questions as you wish, but any marks in excess of 40% will be disregarded.*

1. Starting from the fluid equation for cosmological systems

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0,$$

where ρ is the density, a is the scale factor and P is the pressure, determine how the radiation-, matter- and dark energy- density of the Universe scales with a. **[10]**

2. Starting from the Friedmann equation, show that during a radiation-dominated epoch, the value of the density parameter Ω will approach unity with increasing redshift and explain how this behaviour leads to the flatness problem in our Universe. **[10]**

3. Consider a flat universe with H_0 = 70 km s^{-1} Mpc^{-1} and Ω_M = 1. In this universe, what is the luminosity distance of a galaxy at z=3? If this galaxy has an observed bolometric flux of 3 $\times$ 10^{-19} W m^{-2}, what is its intrinsic bolometric luminosity? **[10]**

4. Describe three pieces of observational evidence for dark matter. Why is the dark matter thought to be non-baryonic? **[10]**

5. Consider a hypothetical universe in which the neutron half-life is 220 seconds rather than the 614 seconds of our Universe. In this hypothetical universe, determine the fraction of the total mass of baryons in the form of Helium-4 at the completion of nucleosynthesis. You may assume that, while in thermal equilibrium, the ratio of neutrons to protons ($\frac{N_n}{N_P}$) is $\frac{1}{5}$ and that the timescale for nuclear reactions to occur following thermal equilibrium is 400 seconds. **[10]**

6. Consider a flat, matter-dominated Universe with Ω_m = 1. Derive an expression for how the matter density ρ_m varies with time. **[10]**

SECTION 2

Answer **one** *question from this Section. If you answer more than one question, credit will only be given for the best answer.*

7. (a) If the mean luminosity density of the Universe in blue light is j_B = 1.7$\times$ 10^8 L$_\odot$ Mpc^{-3}, calculate the present-day total mass-to-light ratio if Ω_M = 1. How does this compare to the mass-to-light ratio of a galaxy like the Milky Way? Assume the Hubble constant H_0 = 70 km s^{-1} Mpc^{-1}. **[8]**

(b) If the present energy in cosmic microwave background radiation is 4.17 $\times$ 10^{-14} J m^{-3}, and the primordial abundance ratio of deuterium and hydrogen (D/H) is found to imply a baryon-photon ratio η = 1 $\times$ 10^{-9}, calculate Ω_b. **[10]**

(c) If primordial black holes of mass 10^{-10} M$_\odot$ make up the dark matter in our galactic halo, calculate on average how far away the nearest such black hole is. State any assumptions you make. Explain whether such black holes would be common or uncommon in the local environment. **[7]**

(d) If the primordial black holes were formed from the collapse of baryonic material, does this conflict with your results from part (b)? Why or why not? **[5]**

8. (a) Using the flux-redshift and angular size-redshift relations, derive an expression for how the surface brightness (i.e. flux per unit solid angle) of a galaxy scales with redshift. The final expression should be in terms of the physical size of the galaxy l, the luminosity of the galaxy L, and the redshift of the galaxy z. **[6]**

   (b) The Hubble Space Telescope can detect objects to a surface brightness (i.e. flux per unit solid angle) limit of 9.1 $L_{\odot}$ pc^{-2} in the V band. Ignoring k-corrections, at what redshift would a uniform circular disc galaxy of radius 8 kpc and an absolute luminosity of 1 $\times$ 10^{10} solar luminosities drop below the surface brightness limit. You may assume that the galaxy is seen 'face-on' to the sky. **[6]**

   (c) Derive an approximate expression, valid in the low redshift limit, for the angular diameter of an object of proper size l at redshift z in a universe with deceleration parameter q_0. In a flat $\Omega_m = 1$ universe with H_0 = 70 km s^{-1} Mpc^{-1}, what is the angular size of the galaxy from part (b) when it becomes undetectable? **[8]**

   (d) Calculate the lookback time of the Universe at the redshift calculated in part (b). **[4]**

   (e) If the flux of a galaxy (f_ν) is a power law (f_ν = $A\ \nu^{-\alpha}$), determine the k-correction for that galaxy at z=2 and α = 1.5. **[6]**

Astrophysical Constants and Units

Astronomical Unit	AU	1.50×10^{11} m
Parsec	pc	3.1×10^{16} m
Tropical year	y	365.242 mean solar days
Solar luminosity	$L_\odot$	3.84×10^{26} W
Absolute bolometric magnitude of the Sun	$M_{\rm bol}$	$+4^{\rm m}75$
Bolometric correction for the Sun	BC	$-0^{\rm m}08$
Apparent visual magnitude of the Sun	$m_v(\odot)$	$-26^{\rm m}74$
Solar constant	f	$1.36 \times 10^{3}\ \mathrm{W\,m^{-2}}$
Solar mass	$M_\odot$	1.989×10^{30} kg
Wien constant	b	2.898×10^{-3} m K
Hubble constant	H_o	$70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$
Solar radius	$R_\odot$	6.95×10^{8} m
Distance of the Sun from galactic centre		8.3 kpc
Earth mass	$M_\oplus$	5.972×10^{24} kg
Earth radius	$R_\oplus$	6371 km

Physical Constants and Units

Acceleration due to gravity	g	9.81 m s^{-2}
Gravitational constant	G	6.674 $\times$ 10^{-11} N m^2 kg^{-2}
Ice point	T_{ice}	273.15 K
Avogadro constant	N_A	6.022 $\times$ 10^{23} mol^{-1}
		[*N.B.* 1 mole $\equiv 1$ *gram-molecule*]
Gas constant	R	8.314 J K^{-1} mol^{-1}
Boltzmann constant	k, k_B	1.381 $\times$ 10^{-23} J K^{-1} $\equiv$ 8.62 $\times$ 10^{-5} eV K^{-1}
Stefan constant	σ	5.670 $\times$ 10^{-8} W m^{-2} K^{-4}
Rydberg constant	R_∞	1.097 $\times$ 10^7 m^{-1}
	$R_\infty hc$	13.606 eV
Planck constant	h	6.626 $\times$ 10^{-34} J s $\equiv$ 4.136 $\times$ 10^{-15} eV s
$h/2\pi$	$\hbar$	1.055 $\times$ 10^{-34} J s $\equiv$ 6.582 $\times$ 10^{-16} eV s
Speed of light *in vacuo*	c	2.998 $\times$ 10^8 m s^{-1}
	$\hbar c$	197.3 MeV fm
Charge of proton	e	1.602 $\times$ 10^{-19} C
Mass of electron	m_e	9.109 $\times$ 10^{-31} kg
Rest energy of electron		0.511 MeV
Mass of proton	m_p	1.673 $\times$ 10^{-27} kg
Rest energy of proton		938.3 MeV
One atomic mass unit	u	1.66 $\times$ 10^{-27} kg
Atomic mass unit energy equivalent		931.5 MeV
Electric constant	ϵ_0	8.854 $\times$ 10^{-12} F m^{-1}
Magnetic constant	μ_0	4π $\times$ 10^{-7} H m^{-1}
Bohr magneton	μ_B	9.274 $\times$ 10^{-24} A m^2 (J T^{-1})
Nuclear magneton	μ_N	5.051 $\times$ 10^{-27} A m^2 (J T^{-1})
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	7.297 $\times$ 10^{-3}= 1/137.0
Compton wavelength of electron	$\lambda_c = h/m_e c$	2.426 $\times$ 10^{-12} m
Bohr radius	a_0	5.2918 $\times$ 10^{-11} m
angstrom	Å	10^{-10} m
barn	b	10^{-28} m^2
torr (mm Hg at 0 °C)	torr	133.32 Pa (N m^{-2})