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Any Calculator
Open Note

UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF BSc & MSci WITH HONOURS

THIRD YEAR EXAMINATION

03 00716

Observational Cosmology

Total time allowed: 1 hour 30 minutes

MAY/JUNE 2010

Answer Section 1 and any one question from Section 2.

Section 1 counts for 40% of the marks for the examination. Full marks may be obtained by correctly answering four questions. You may attempt as many questions as you wish, but any marks in excess of 40% will be disregarded.

Section 2 consists of two questions, each carrying 30% of the marks. If you answer more than one question from this Section, only the first will be marked.

A further 30% of the course credit derives from coursework assignments already submitted. The approximate allocation of marks to each part of a question is shown in brackets [ ].

Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

Students may use their course notes in the examination, but textbooks are not permitted

A table of physical constants and units that may be required will be found at the end of this question paper.

SECTION 1

Full marks may be obtained by correctly answering four questions. You may attempt as many questions as you wish, but any marks in excess of 40% will be disregarded.

1. Consider a universe with $k = +10$. Show that by simply rescaling the scale factor and the radial coordinate of the Friedmann-RobertsonWalker metric by appropriate constant factors, you can turn this universe into one characterised by $k = +1$. **[10]**

2. Consider a universe where the value of the cosmological constant is zero. Show that regardless of the actual matter, radiation and curvature content, at sufficiently early cosmic times t the scale factor a of such a universe scales as $a(t) \propto t^{1/2}$. **[10]**

3. In the early Universe, neutrons and protons remain in thermal equilibrium until the Universe cools sufficiently that the interaction rate is smaller than the expansion rate. This happens at an energy $k_B T \simeq 0.8$ MeV, corresponding to a cosmic time $t \approx 400$ s (which is comparable to the neutron's half-life ≈ 600 s). Evaluate the neutron-to-proton ratio at the time of cosmic nucleosynthesis. **[10]**

4. Show that the number density of photons today overwhelms the number density of baryons. **[10]**

5. Consider a universe where the value of the cosmological constant Λ is zero. What is the value of the curvature parameter k for which such a universe ultimately ends up in a "Big Crunch"? Justify your answer. **[10]**

6. Explain what is meant by the K-correction. For what form of the spectrum of a source does the K-correction vanish. **[10]**

SECTION 2

You should attempt one question from this Section. If you answer more than one question, only the first one will be marked.

7. (a) Consider the process of structure formation after decoupling. Assume that a region of size L of the Universe (at temperature T) is made up of identical particles of mass m at uniform density ρ. By considering the balance between gravitational and thermal energy show that the system will collapse for $L > L_J$, where L_J is the Jeans length. Derive an expression of L_J as a function of T, ρ and m. **[10]**

(b) Hence, estimate the minimum mass that collapses after decoupling and the associated collapse timescale. **[12]**

(c) By considering the density evolution of the Universe shortly after decoupling, determine how the collapse timescale depends on the cosmic time. **[8]**

8. (a) Estimate of the temperature T at which the cosmic microwave background was "generated" by considering the mean photon energy at a given temperature and the ionisation energy of hydrogen. **[10]**

(b) The previous analysis yields a value for the temperature that differs by a factor of ≈ 7 from the actual value. Explain why this is so, and whether the previous result overestimates or underestimates the actual temperature at decoupling. **[5]**

(c) Considering the ratio of baryons to photons and the energy distribution of photons, revise the estimate made in part (a) to derive a more precise value for the temperature at decoupling and comment on its relation to the actual temperature. **[15]**

Physical Constants and Units

(For all years)

Acceleration due to gravity	g	$9.81\ \mathrm{ms^{-2}}$
Gravitational constant	G	$6.673 \times 10^{-11}\ \mathrm{Nm^{-2}kg^{-2}}$
Avogadro constant	N_A	$6.022 \times 10^{23}\ \mathrm{mol^{-1}}$
	Note: 1 mole = 1 *gram* molecular-weight	
Ice point	T_ice	273.15 K
Gas Constant	R	$8.314\ \mathrm{JK^{-1}mol^{-1}}$
Boltzmann constant	k, k_B	$1.381 \times 10^{-23}\ \mathrm{JK^{-1}} = 0.862 \times 10^{-4}\ \mathrm{eV\ K^{-1}}$
Stefan constant	σ	$5.670 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$
Rydberg constant	R_∞	$1.097 \times 10^{7}\ \mathrm{m^{-1}}$
	$R_\infty hc$	$13.61\ \mathrm{eV}$
Planck constant	h	$6.626 \times 10^{-34}\ \mathrm{Js} = 4.136 \times 10^{-15}\ \mathrm{eV\ s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34}\ \mathrm{Js} = 6.582 \times 10^{-16}\ \mathrm{eV\ s}$
Speed of light *in vacuo*	c	$2.998 \times 10^{8}\ \mathrm{ms^{-1}}$
	$\hbar c$	$197.3\ \mathrm{MeV\ fm}$
Charge of proton	e	$1.602 \times 10^{-19}\ \mathrm{C}$
Mass of electron	m_e	$9.109 \times 10^{-31}\ \mathrm{kg}$
Rest energy of electron		$0.511\ \mathrm{MeV}$
Mass of proton	m_p	$1.673 \times 10^{-27}\ \mathrm{kg}$
Rest energy of proton		$938.3\ \mathrm{MeV}$
One atomic mass unit	u	$1.660 \times 10^{-27}\ \mathrm{kg}$
Atomic mass unit energy equivalent		$931.5\ \mathrm{MeV}$
Electric constant	ε_0	$8.854 \times 10^{-12}\ \mathrm{Fm^{-1}}$
Magnetic constant	μ_0	$4\pi \times 10^{-7}\ \mathrm{Hm^{-1}}$
Bohr magneton	μ_B	$9.274 \times 10^{-24}\ \mathrm{Am^2}\ (\mathrm{JT^{-1}})$
Nuclear magneton	μ_N	$5.051 \times 10^{-27}\ \mathrm{Am^2}\ (\mathrm{JT^{-1}})$
Fine structure constant	$\alpha = e^2/4\pi\varepsilon_0\hbar c$	$7.297 \times 10^{-3} = 1/137.0$
Compton wavelength of electron	$\lambda_\mathrm{C} = h/mc$	$2.426 \times 10^{-12}\ \mathrm{m}$
Bohr radius	a_0	$5.292 \times 10^{-11}\ \mathrm{m}$
angstrom	Å	$10^{-10}\ \mathrm{m}$
torr (mmHg, 0°C)	torr	$133.3\ \mathrm{Pa}\ (\mathrm{Nm^{-2}})$
barn	b	$10^{-28}\ \mathrm{m^2}$

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