UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF BSc & MSci WITH HONOURS

THIRD YEAR EXAMINATION

03 00716

Observational Cosmology

Total time allowed: 1 hour 30 minutes

MAY/JUNE 2011

Answer Section 1 and any one question from Section 2.

Section 1 counts for 40% of the marks for the examination. Full marks may be obtained by correctly answering four questions. You may attempt as many questions as you wish, but any marks in excess of 40% will be disregarded.

Section 2 consists of two questions, each carrying 30% of the marks. If you answer more than one question from this Section, only the first will be marked.

The approximate allocation of marks to each part of a question is shown in brackets [ ]. A further 30% of the course credit derives from coursework assignments already submitted.

Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

Students may use their course notes in the examination, but textbooks are not permitted

A table of physical constants and units that may be required will be found at the end of this question paper.

SECTION 1

Full marks may be obtained by correctly answering four questions. You may attempt as many questions as you wish, but any marks in excess of 40% will be disregarded.

1. A *de Sitter* universe is a cosmological model in which the universe is empty and flat. For this model, show that the scale factor grows exponentially and that the Hubble parameter is constant. **[10]**

2. Consider a universe where the cosmological constant is zero. Considering arbitrary contributions of matter, radiation and curvature, show that at sufficiently early cosmic times t, the scale factor a of such a universe scales as $a(t) \propto t^{1/2}$. **[10]**

3. Consider an over-dense region of the Universe just after decoupling, whose mass density is $2M_{\odot}\mathrm{pc}^{-3}$. Estimate the mass of the first halo to collapse in this region. **[10]**

4. The observed dipole anisotropy in the CMB maps corresponds to temperature differences of the order $\Delta T/T \sim 10^{-3}$. Estimate the velocity of our galaxy with respect to a frame in which the CMB is isotropic and show that the angular dependence of $\Delta T/T$ is approximately sinusoidal with the angle on the sky. **[10]**

5. By considering the ionizing potential of the ground state of hydrogen, estimate the decoupling temperature of the photons from the cosmic plasma. Discuss whether (and why) this differs from its actual value. **[10]**

6. Show quantitatively that a spatially flat universe requires fine tuning of the initial conditions. **[10]**

SECTION 2

You should attempt one question from this Section. If you answer more than one question, only the first one will be marked.

7. (a) Derive the present day number density of photons $n_{\gamma,0}$ in the cosmic microwave background, and the corresponding density parameter $\Omega_{\gamma,0}$. Taking $\Omega_{r,0} = 4.15 \times 10^{-5} h^{-2}$ for the value of the density parameter of radiation today, explain why $\Omega_{\gamma,0}$ differs from $\Omega_{r,0}$. [Here h is the Hubble constant in units of 100 $\mathrm{km\,s^{-1}Mpc^{-1}}$]. **[12]**

   (b) Using the above value of $\Omega_{r,0}$, compute the ratio of the scale factor today to that at the epoch of matter-radiation equality. **[10]**

   (c) Evaluate the temperature of the cosmic plasma at the epoch of matter-radiation equality and discuss whether at that epoch the Universe was opaque to photons. **[8]**

8. (a) The rotation velocity of the disk galaxy NGC3198 is measured to be $\approx 150\,\mathrm{km\,s^{-1}}$ at a distance $r = 8$ kpc from its centre. Estimate the mass of the galaxy within that radius and discuss the validity of the assumptions you have made. **[12]**

(b) It has been suggested that the dark matter in the halos of galaxies could be accounted for by a population of $\sim 10^{-10} M_\odot$ black holes. Assuming that this is indeed the case, estimate the distance from us of the nearest such black hole, and compare it to the typical separation between stars. **[10]**

(c) Adopting the current consensus cosmology, what shall the values of Ω_m and Ω_Λ be when the scale factor has increased by a factor of 10. **[8]**

Physical Constants and Units

(For all years)

Acceleration due to gravity	g	$9.81\ \mathrm{ms^{-2}}$
Gravitational constant	G	$6.673 \times 10^{-11}\ \mathrm{Nm^{-2}kg^{-2}}$
Avogadro constant	N_A	$6.022 \times 10^{23}\ \mathrm{mol^{-1}}$
	Note: 1 mole = 1 *gram* molecular-weight	
Ice point	T_ice	273.15 K
Gas Constant	R	$8.314\ \mathrm{JK^{-1}mol^{-1}}$
Boltzmann constant	k, k_B	$1.381 \times 10^{-23}\ \mathrm{JK^{-1}} = 0.862 \times 10^{-4}\ \mathrm{eV\ K^{-1}}$
Stefan constant	σ	$5.670 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$
Rydberg constant	R_∞	$1.097 \times 10^{7}\ \mathrm{m^{-1}}$
	$R_\infty hc$	$13.61\ \mathrm{eV}$
Planck constant	h	$6.626 \times 10^{-34}\ \mathrm{Js} = 4.136 \times 10^{-15}\ \mathrm{eV\ s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34}\ \mathrm{Js} = 6.582 \times 10^{-16}\ \mathrm{eV\ s}$
Speed of light *in vacuo*	c	$2.998 \times 10^{8}\ \mathrm{ms^{-1}}$
	$\hbar c$	$197.3\ \mathrm{MeV\ fm}$
Charge of proton	e	$1.602 \times 10^{-19}\ \mathrm{C}$
Mass of electron	m_e	$9.109 \times 10^{-31}\ \mathrm{kg}$
Rest energy of electron		$0.511\ \mathrm{MeV}$
Mass of proton	m_p	$1.673 \times 10^{-27}\ \mathrm{kg}$
Rest energy of proton		$938.3\ \mathrm{MeV}$
One atomic mass unit	u	$1.660 \times 10^{-27}\ \mathrm{kg}$
Atomic mass unit energy equivalent		$931.5\ \mathrm{MeV}$
Electric constant	ε_0	$8.854 \times 10^{-12}\ \mathrm{Fm^{-1}}$
Magnetic constant	μ_0	$4\pi \times 10^{-7}\ \mathrm{Hm^{-1}}$
Bohr magneton	μ_B	$9.274 \times 10^{-24}\ \mathrm{Am^2}\ (\mathrm{JT^{-1}})$
Nuclear magneton	μ_N	$5.051 \times 10^{-27}\ \mathrm{Am^2}\ (\mathrm{JT^{-1}})$
Fine structure constant	$\alpha = e^2/4\pi\varepsilon_0\hbar c$	$7.297 \times 10^{-3} = 1/137.0$
Compton wavelength of electron	$\lambda_\mathrm{C} = h/mc$	$2.426 \times 10^{-12}\ \mathrm{m}$
Bohr radius	a_0	$5.292 \times 10^{-11}\ \mathrm{m}$
angstrom	Å	$10^{-10}\ \mathrm{m}$
torr (mmHg, 0°C)	torr	$133.3\ \mathrm{Pa}\ (\mathrm{Nm^{-2}})$
barn	b	$10^{-28}\ \mathrm{m^2}$

END OF PAPER