ANY CALCULATOR

UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF BSc & MSci WITH HONOURS

FOURTH YEAR FINAL EXAMINATION

03 14988

Formation and Evolution of galaxies

Total time allowed: 2 Hours

SUMMER 2012

Answer Section 1 and any two questions from Section 2.

Section 1 counts for 40% of the marks for the examination. Full marks may be obtained by correctly answering four questions. You may attempt as many questions as you wish, but any marks in excess of 40% will be disregarded.

Section 2 consists of three questions and carries 60% of the marks (30% for each question). If you answer more than two questions from this Section, only the first two will be marked.

The approximate allocation of marks to each part of a question is shown in brackets [ ].

Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

A table of physical constants and units that may be required will be found at the end of this question paper.

SECTION 1

Full marks may be obtained by correctly answering four questions. You may attempt as many questions as you wish, but any marks in excess of 40% will be disregarded.

1. A globular cluster, consisting of 2.3×10^5 identical stars of the same mass as the Sun, has a velocity dispersion $\sigma = 10$ km s^{-1}. Its angular radius on the sky is 3 arcmin. What is its distance from us? **[10]**

2. Explain the "winding paradox" in the context of spiral galaxies (with a suitable figure) and qualitatively outline how it can be resolved. **[10]**

3. Describe the principal differences between spiral and elliptical galaxies with respect to (a) their constituents and (b) evolutionary history. Which of these types of galaxies would you predominantly expect to find in the cores of clusters of galaxies, and why? **[10]**

4. The abundance ratio of oxygen to iron [O/Fe] is found to be approximately constant with the metallicity [Fe/H] for metal-deficient stars. However, for metal-rich stars, it is found to decrease with increasing metallicity. Explain why this is so. What would you expect the value of the abundance [O/Fe] to be for stars with iron abundance [Fe/H]=0? **[10]**

5. The present average star formation rate within the star-forming region of the disk of the Milky Way galaxy is believed to be $5.0 \pm 0.5\, M_{\odot}$ pc^{-2} Gyr^{-1}. Using reasonable values for the required parameters, estimate the number of stars being formed each year. Given that the stellar mass of the Milky Way is $2 \times 10^{10} M_{\odot}$, comment on the star formation history of the Galaxy. **[10]**

6. Describe the observational differences between Seyfert I and Seyfert II galaxies. Explain how these relate to the unified model of active galactic nuclei. Use a schematic figure if necessary. **[10]**

SECTION 2

You should attempt one questions from this Section. If you answer more than one question, only the first will be marked.

7. A spherically symmetric galaxy has a dark matter halo whose density depends on the radial distance r as

$$\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha},$$

where ρ_0, r_0 and α are constants.

(a) Derive an expression for the total mass $M(R)$ of the dark matter halo within a radius R. What are the constraints on α in order to ensure that M remains finite for finite R? **[6]**

(b) Show that the gravitational potential for this system is

$$\Phi(r) = \frac{4\pi G \rho_0 r_0^2}{(2-\alpha)(3-\alpha)} \left(\frac{r}{r_0}\right)^{2-\alpha}.$$

[8]

(c) Find an expression for the circular velocity $v_c(r)$ in this dark halo. What value of α is necessary to produce a flat rotation curve at large values of r? **[8]**

(d) Show that for values of the power-law index in the range $2 < \alpha < 3$, even though the mass $M(r)$ diverges at large r, the value of the escape velocity $v_e(r)$ remains finite. Comment on the behaviour of the ratio $(v_e/v_c)^2$ as a function of α in this range. **[8]**

8. A gas cloud of density ρ and sound speed c_s collapses under its own gravity.

   (a) Write down an expression for the dispersion relation for acoustic waves in such a cloud. Hence derive the Jeans length, and the corresponding Jeans mass, of the cloud in terms of the given quantities. **[8]**

   (b) What is the physical significance of the Jeans length and mass? Give a qualitative explanation of the phenomenon in terms of energy. **[4]**

   (c) Given that the speed of sound c_s is related to the pressure p and density ρ of the cloud by $c_s^2 = dP/d\rho$, show how the Jeans mass depends on the temperature of the Universe in the (i) radiation dominated era $t < t_{\rm eq}$, and (ii) after recombination, when matter decouples from radiation. **[12]**

   (d) Comment on the relative characteristic masses that emerge from the above calculation, and its implication for galaxy formation. **[6]**

SECTION 3

This question is compulsory for Year 4 students.

9. (a) Outline the observational evidence (present at least three independent arguments) for the presence of supermassive black holes at the centres of galaxies. **[6]**

(b) The maximum possible angular momentum of an electrically neutral, rotating black hole of mass M is

$$L_{\max} = \frac{GM^2}{c}.$$

Assuming the Schwarzschild radius to define the size of a black hole, calculate the values of (i) the average density, (ii) surface gravity and (iii) the maximum angular velocity $\omega_{\max}$ and (iv) the corresponding linear velocities of a $10^8 M_{\odot}$ black hole. Compare these values with those for the Sun. Use Newtonian physics to make these calculations, and point out the limitation of this approach. **[10]**

(c) Using Newtonian physics, show that the luminosity of an accretion disk, with an inner radius R around a star of mass M is given by

$$L_{\text{disk}} = G\frac{M\dot{M}}{2R},$$

where $\dot{M}$ is the rate of accretion of matter on to the disk.

Applying this to an accretion disk around a non-rotating black hole, calculate the efficiency of accretion luminosity for the innermost stable circular orbit around this black hole. **[7]**

(d) In view of these calculations, write a short account of how asupermassive black hole can be a significant source of energy responsible for heating the interstellar and intergalactic medium surrounding it. In what parts of the electromagnetic spectrum will this energy output be seen, and in what form? **[7]**

Physical Constants and Units

(For all years)

Acceleration due to gravity	g	$9.81\ \mathrm{ms^{-2}}$
Gravitational constant	G	$6.673 \times 10^{-11}\ \mathrm{Nm^{-2}kg^{-2}}$
Avogadro constant	N_A	$6.022 \times 10^{23}\ \mathrm{mol^{-1}}$
	Note: 1 mole = 1 *gram* molecular-weight	
Ice point	T_{ice}	273.15 K
Gas Constant	R	$8.314\ \mathrm{JK^{-1}mol^{-1}}$
Boltzmann constant	k, k_B	$1.381 \times 10^{-23}\ \mathrm{JK^{-1}} = 0.862 \times 10^{-4}\ \mathrm{eV\ K^{-1}}$
Stefan constant	σ	$5.670 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$
Rydberg constant	R_∞	$1.097 \times 10^{7}\ \mathrm{m^{-1}}$
	$R_\infty hc$	$13.61\ \mathrm{eV}$
Planck constant	h	$6.626 \times 10^{-34}\ \mathrm{Js} = 4.136 \times 10^{-15}\ \mathrm{eV\ s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34}\ \mathrm{Js} = 6.582 \times 10^{-16}\ \mathrm{eV\ s}$
Speed of light *in vacuo*	c	$2.998 \times 10^{8}\ \mathrm{ms^{-1}}$
	$\hbar c$	$197.3\ \mathrm{MeV\ fm}$
Charge of proton	e	$1.602 \times 10^{-19}\ \mathrm{C}$
Mass of electron	m_e	$9.109 \times 10^{-31}\ \mathrm{kg}$
Rest energy of electron		$0.511\ \mathrm{MeV}$
Mass of proton	m_p	$1.673 \times 10^{-27}\ \mathrm{kg}$
Rest energy of proton		$938.3\ \mathrm{MeV}$
One atomic mass unit	u	$1.660 \times 10^{-27}\ \mathrm{kg}$
Atomic mass unit energy equivalent		$931.5\ \mathrm{MeV}$
Electric constant	ε_0	$8.854 \times 10^{-12}\ \mathrm{Fm^{-1}}$
Magnetic constant	μ_0	$4\pi \times 10^{-7}\ \mathrm{Hm^{-1}}$
Bohr magneton	μ_B	$9.274 \times 10^{-24}\ \mathrm{Am^2}\ (\mathrm{JT^{-1}})$
Nuclear magneton	μ_N	$5.051 \times 10^{-27}\ \mathrm{Am^2}\ (\mathrm{JT^{-1}})$
Fine structure constant	$\alpha = e^2/4\pi\varepsilon_0 \hbar c$	$7.297 \times 10^{-3} = 1/137.0$
Compton wavelength of electron	$\lambda_C = h/mc$	$2.426 \times 10^{-12}\ \mathrm{m}$
Bohr radius	a_0	$5.292 \times 10^{-11}\ \mathrm{m}$
angstrom	Å	$10^{-10}\ \mathrm{m}$
torr (mmHg, 0°C)	torr	$133.3\ \mathrm{Pa}\ (\mathrm{Nm^{-2}})$
barn	b	$10^{-28}\ \mathrm{m^2}$