Part IV
Spiral Galaxies

Introduction to Spiral Galaxies

The characteristic feature of spiral galaxies is that they have a disk-like appearance with well-defined spiral arms emanating from their central regions.

They often have central bars and/or rings.

Often the spiral pattern has a remarkable degree of symmetry with respect to the centre of the galaxy.

The light distribution of a 'normal' spiral galaxy is made up of:

1. a *central bulge* or *spheroid*, similar to an elliptical galaxy,
2. a disk component, in which the spiral arms lie.

The luminosity of the disk of the spiral can be represented as

$$L(R, z) = L_0 \exp\left(-\frac{R}{R_0}\right) \text{sech}^2\left(\frac{z}{z_0}\right), \tag{4.1}$$

where R and z are distances measured in the radial direction from the centre of the galaxy and perpendicular to the disk respectively.

R_0 and z_0 set the scale-lengths in both directions, and L_0 is the central luminosity.

Very roughly, the starlight comes from a region with height about 10% of the width.

The gas and dust have scale heights smaller than that of stars.

In contrast, the luminosity of the bulge follows a de Vaucouleurs profile (see *Part VI*).

The central surface brightness of larger spirals is found to be reasonably constant, with a B-band magnitude of $\sim 21.65 \pm 0.3$ mag/arcsec^2 (Freeman's law).

The Tully-Fisher (TF) Relation for Disk galaxies

We can use either radio or optical/IR measurements to determine the peak rotation velocity V_{Max} in the disk of galaxies.

Bigger more luminous galaxies rotate more rapidly than smaller galaxies, with $L \sim V_{Max}^{\alpha}$, with $\alpha \sim 4$.

TF relationship works best in the R-band or IR portion of spectrum.

Blue/UV light is much more affected by bursts of recent star-formation (with V_{max} unaffected).

In the H-band (1.65μm) we have

$$\frac{L_H}{3 \times 10^{10} L_{H,\odot}} = \left(\frac{V_{Max}}{196 \text{ km s}^{-1}}\right)^4 \tag{4.2}$$

The Tully-Fisher relationship in largely an empirical relationship.

V_{max} largely determined by dark matter.

Existence of TF relationship means the amount of dark matter is related to amount of luminous matter in disk galaxies – not obvious.

TF relationship is an important component in cosmic distance measurements.

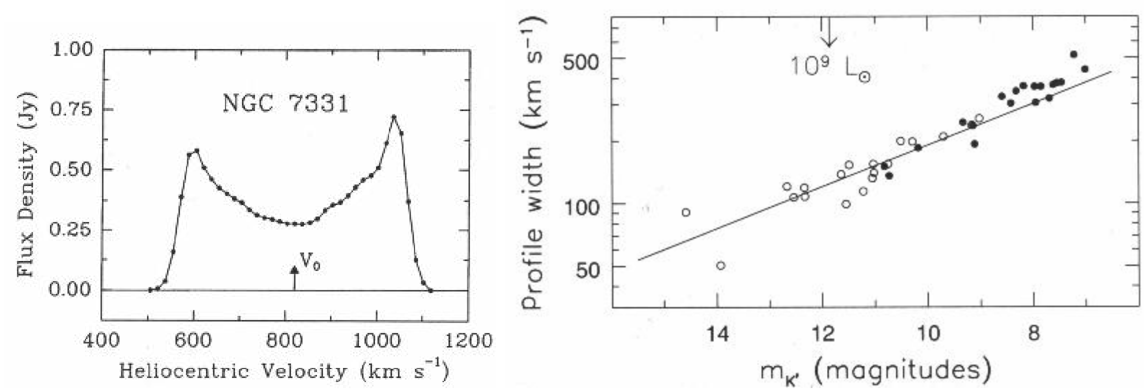

Figure 4.1: Left panel: HI global profile for a disk galaxy (NGC 7331), showing the horned profile that can be used to determine V_{Max}. The separation of the peaks is $W = 2V_{Max}\sin i$, with i the inclination of the galaxy. Right panel: the Tully-Fisher relationship for disk galaxies in the Ursa Major group of galaxies, as seen in the IR (K-band, at 2.2μm).

The Winding Dilemma for Spiral Arms

Spiral arms are seen in almost all disk galaxies, but they do not appear to be very tightly wound up, even though the disk is rotating.

If the disk of a spiral rotates with an angular speed $\Omega(R)$, and at any given epoch a radial stripe is drawn across its disk, the equation for the stripe can be written as

$$\phi(R,t) = \phi_0 + \Omega\,(R)\,t. \tag{4.3}$$

The pitch angle i is defined as the angle between the tangent to the arm and the circle R =constant:

$$\cot i = \left| R\frac{d\phi}{dR} \right|. \tag{4.4}$$

Imagine that the spiral arms are being wound up due to the rotation of the disk.

If the nearest successive location of arms at azimuth ϕ are at R and $R + \Delta R$, then

$$2\pi = \left| \Delta R\frac{d\phi}{dR} \right|, \tag{4.5}$$

since one winding corresponds to the change in ϕ of 2π.

If $\Delta R \ll R$ (i small), $\Delta R = 2\pi R/\cot i = 2\pi R\tan i$.

Typical values for a star at location of Sun: $v_c = 220$ km/s, $R = 10$ kpc, $t = 10^{10}$ yr.

We have $\Omega \sim v_c/R$, so that

$$\frac{d\phi}{dR} = \left(\frac{d\Omega}{dR}\right)t = -\frac{v_c t}{R^2} \tag{4.6}$$

Hence, we find $i = 0.25$ degrees and $\Delta R = 0.28$ kpc.

This corresponds to a spiral arm that is far too tightly wound compared to what is observed.

Observed pitch angles are about $\sim$ 10 degrees or so, and we never see spiral arms which are wound up more than a few times around.

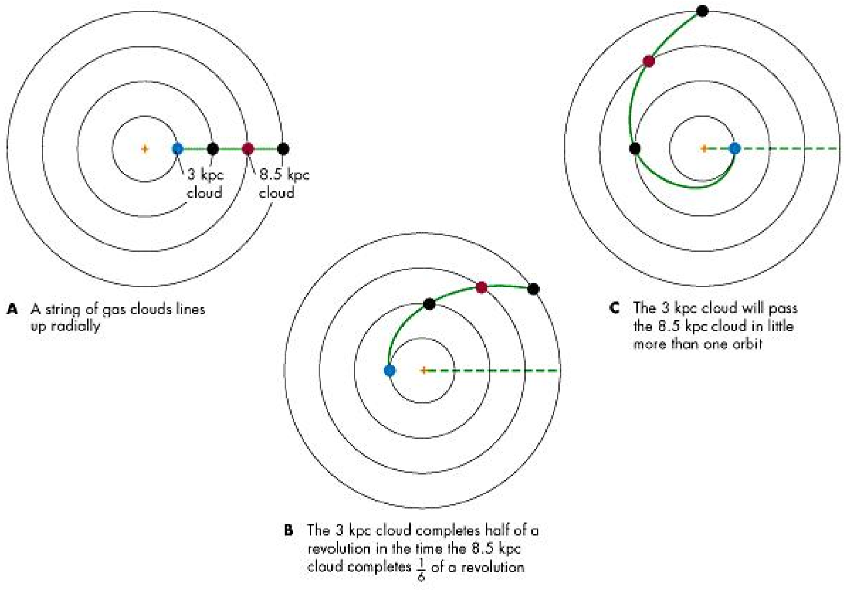

Figure 4.2: The winding dilemma in spiral galaxies. Note we assume that all clouds move with the same velocity v (flat portion of rotation curve), so that the angular velocity $\Omega \sim v/R$, with R the distance from the galaxy centre.

For Sa galaxies: $i \sim 5°$. For Sc: $10° < i < 30°$ (basis of Sa/Sb/Sc classification).

The most likely implication is that spiral arms are not material features.

The winding dilemma arises from thinking of spiral arms as material alignments in a differentially rotating disk.

The way out of this dilemma is that the spiral structure is a (density) wave phenomenon, maintained by the self-gravity of the distribution of matter in the disk, so that at different times, the density enhancement seen at a given place is made up of different stars/clouds.

Theories of Spiral Structure

That rotating disk galaxies should exhibit spiral structure is not surprising, but the nature of spiral structure is still not completely understood.

Spiral patterns in disk galaxies can arise from various sources.

- Kinematic spiral waves are perturbations that naturally arise in a differentially rotating system.
- Spiral waves can also be caused by tidal interaction with neighbours.
- Example: Water molecules in the ocean do not move very far in response to a passing wave.
- Similarly, stars in a disk galaxy need not move far from their unperturbed orbits to create a spiral density wave.

In disk galaxies, most stars are on nearly circular orbits, so it is useful to consider their orbits as basically circular, but with small perturbations in the R and z directions.

Spiral Structure: Epicycles

Recall our discussion on the *effective potential* in the case of central forces.

Consider a spiral galaxy to be an axisymmetric system, and write the general equation of motion of a star in such a galaxy in cylindrical polar coordinates (R, ϕ, z).

Consider motion within a galaxy potential Φ.

The three components of acceleration are given by

$$\ddot{R} - R\dot{\phi}^2 = -\frac{\partial \Phi}{\partial R}, \tag{4.7}$$

$$\frac{d}{dt}\left(R^2\dot{\phi}\right) = 0, \tag{4.8}$$

$$\ddot{z} = -\frac{\partial \Phi}{\partial z}. \tag{4.9}$$

The second of these equations (as before) provides us with the law of conservation of angular momentum $L_z = R^2\dot{\phi}$ =constant.

A star with angular momentum L_z can follow an exactly circular orbit only at the radius R_g where the effective potential is stationary with respect to R, such that

$$\frac{\partial \Phi_{\text{eff}}}{\partial R} = 0 = \frac{\partial}{\partial R}\left[\Phi(R, z) + \frac{L_z^2}{2R^2}\right]. \tag{4.10}$$

At $R = R_g$, $z = 0$ (in the meridional plane), this means

$$\left.\frac{\partial \Phi}{\partial R}\right|_{R_g, z=0} = -\frac{\partial}{\partial R}\left(\frac{L_z^2}{2R^2}\right) = \frac{L_z^2}{R_g^3} = R_g\Omega^2(R_g), \tag{4.11}$$

since $\Omega = \dot{\phi}(R_g) = L_z/R_g^2$ from its definition.

If Φ_{eff} is minimum at $R = R_g$, the corresponding circular orbit has minimum energy for given L_z, and so is stable.

Any star with same L_z will oscillate about this mean orbit, with small perturbations in the radial and z directions.

As the star moves radially in and out, its azimuthal motion must alternately speed up and slow down respectively.

Therefore such a star would follow an approximately elliptical "epicycle" around the *guiding centre* R_g, which moves with angular speed $\Omega(R_g)$ in a circular orbit.

If x and y are coordinates in a 'not-quite-Cartesian' frame of reference revolving about the centre of the galaxy with angular velocity Ω of a circular orbit at radius $R{=}R_g$.

In terms of polar coordinates in the plane of the disk,

$$x = R - R_g; \qquad y = R_g(\phi - \Omega t), \tag{4.12}$$

where x increases outward from the centre and y increases in the direction of rotation.

The equation of motion in the radial direction is given by

$$\ddot{R} = \frac{d^2}{dt^2}(R_g + x) = \ddot{x} = -\frac{\partial \Phi_{\text{eff}}}{\partial R} \tag{4.13}$$

Using a Taylor expansion, we have

$$-\ddot{x} = \frac{\partial \Phi_{\text{eff}}}{\partial R} = \left[\frac{\partial \Phi_{\text{eff}}}{\partial R}\right]_{R_g} + (R - R_g)\left[\frac{\partial^2 \Phi_{\text{eff}}}{\partial^2 R}\right]_{R_g} + O(x^2) \tag{4.14}$$

as $(R - R_g) = x$.

Note, that Φ_{eff} is a minimum at $R = R_g$.

Neglecting the higher powers of the small perturbation x, we have

$$\ddot{x} = -\left[\frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2}\right]_{R_g} x = -\kappa^2(R_g)\, x, \tag{4.15}$$

which is the equation for a simple harmonic solution (SHM).

This SHM equation has a solution:

$$x = X\, \cos\left(\kappa t + \psi\right). \tag{4.16}$$

When $\kappa^2 > 0$, this equation describes a simple harmonic motion with frequency κ.

If $\kappa^2 < 0$, the orbit is unstable, and the star moves away from the centre of force.

Since L_z is conserved, and the angular speed of the corresponding circular orbit $\Omega = L_z/R_g^2$, the rate of change of the azimuthal angle of the star is given by

$$\dot{\phi} = \frac{L_z}{R^2} = \frac{L_z}{R_g^2}\left(1 + \frac{x}{R_g}\right)^{-2} \sim \Omega\left(1 - \frac{2x}{R_g}\right). \tag{4.17}$$

Substituting from eqn. (4.16), we have

$$\dot{\phi} = \Omega\left[1 - \frac{2X}{R_g}\cos(\kappa t + \psi)\right]. \tag{4.18}$$

Integrating equation (4.18) with respect to t,

$$\phi = \Omega t + \phi_0 - \frac{2\Omega X}{\kappa R_g}\sin(\kappa t + \psi), \tag{4.19}$$

such that the tangential displacement

$$y = -\frac{2\Omega X}{\kappa}\sin(\kappa t + \psi) = -Y\sin(\kappa t + \psi). \tag{4.20}$$

Eqns. (4.16) and (4.20) together give the complete solution to the orbit of the epicycle the star describes in the plane of the galaxy over and above its circular orbit.

If $X \neq Y$, this epicycle is elliptical, with ratio of axes

$$\frac{X}{Y} = \frac{\kappa}{2\Omega}. \tag{4.21}$$

For a harmonic oscillator potential (uniform density, solid body rotation), $X/Y = 1$.

However, for a Kepler potential, $X/Y = 1/2$.

In general, $Y \geq X$, so that the epicycle ellipse is elongated in the tangential direction.

From equation 4.15, the radial frequency κ is

$$\kappa^2 = \left[\frac{\partial^2 \Phi_{eff}}{\partial R^2}\right]_{R_g}. \tag{4.22}$$

From this expression, it is easy to show that for a Kepler case (point mass at the centre), $\kappa = \Omega$, whereas in the case of solid-body rotation, $\kappa = 2\Omega$.

The potential of our Galaxy is in between these two cases, so at the position of the Sun,

$$\Omega < \kappa < 2\Omega. \tag{4.23}$$

In fact, the measured value at the Sun shows $\kappa \sim 1.4\Omega$.

Thus, the Sun and nearby disk stars are moving on epicycles which are squashed by about 30% in the radial direction.

The orbit of the Sun thus does not close on itself, and the period of the epicycle is 170 Myr.

Epicycles: Vertical Motion

The motion in the perpendicular direction has a similar solution

$$z = Z\,\cos(\nu t + \psi), \tag{4.24}$$

where the *vertical frequency* ν is given by

$$\nu^2 = \left[\frac{\partial^2 \Phi_{\rm eff}}{\partial z^2}\right]_{z=0}. \tag{4.25}$$

For a highly flattened system (i.e. a spiral disk), we have

$$\frac{\partial^2 \Phi_{\rm eff}(R,z)}{\partial z^2} = 4\pi G\rho(R,z) \tag{4.26}$$

where $\rho(R,z)$ is the mass density. In the solar neighbourhood $\rho(R,z) = 0.15 M_\odot$ /pc^3.

The vertical period $P_{vert} \sim 70$ Myr.

Spiral Structure

We can apply epicycles in constructing kinematic spiral waves.

For example, consider a ring of test particles on similar epicyclic orbits with their guiding centres at the same radius R_g.

Let the initial phases of the epicycles be such that at $t = 0$ the particles describe an oval.

With time the guiding centres travel around the galaxy with angular velocity Ω, but the stars at the ends of the oval are being carried backward with respect to their guiding centres, so the form of the oval advances more slowly.

The rate of precession or "pattern speed" of the oval is given by

$$\Omega_p = \Omega - \frac{\kappa}{2}. \tag{4.27}$$

By superposing ovals of different sizes, we can produce a variety of spiral patterns.

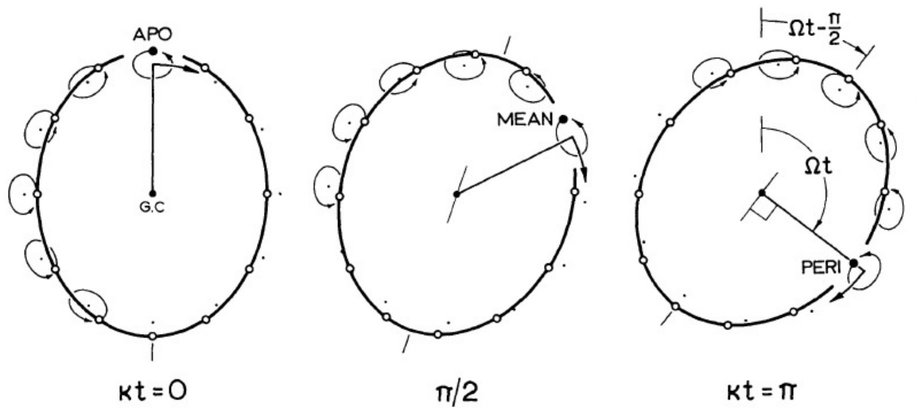

Figure 4.3: The motion of test particles, all moving clockwise with an angular speed Ω, and which are also moving on small counter-clockwise elliptical "epicycles". The motion is for the case $\kappa = \sqrt{2}\Omega$, which is close to the case for the galaxy. The locus of the oval advances considerably more slowly that the particles themselves. The pattern speed of the oval Ω_p is equal to $\Omega - \kappa/2$, and can be verified by comparing the first and last frames (taken from Toomre 1977).

If $\Omega - \kappa/2$ were independent of radius, such patterns would persist indefinitely because all the superposed ovals would precess at the same speed.

For a wide range of plausible disk galaxy models, $\Omega - \kappa/2$ is found to be fairly constant over a large range of radii.

Compared to material arms, density waves in our Galaxy would wind up six times less rapidly, yielding predicted pitch angles of about 1.4 degrees.

This is an improvement, but still not consistent with observations

This simple-minded model has neglected the self-gravity of spiral structure – so it cannot be telling the whole story (see "swing amplification").

In general, for an m-armed spiral, the pattern speed is given by

$$\Omega_p = \Omega - \frac{\kappa}{m}, \tag{4.28}$$

such that stars orbiting at radius r pass though an arm of an m-armed spiral with frequency $m[\Omega_p - \Omega(r)]$.

Spiral pattern persist if $m|\Omega_p - \Omega(r)| < \kappa(r)$, i.e. in the region between $\Omega_p = \Omega \pm \kappa/m$, the *inner* and *outer Lindblad resonances.*

Since κ is usually bounded below by Ω (Keplerian value) and above by 2Ω (solid-body-rotation value), the $m = 1$ (one-armed spiral) and $m = 2$ (two-armed spiral) disturbances may have no inner Lindblad resonance if Ω_p has a sufficiently large positive value.

Barred Disks

Around half of all spirals shows a bar structure (NGC1365 being perhaps the most impressive).

Bars are not fully understood – why some galaxies have bars and others do not.

The bars can be long/thin with a length ratio of up to 5:1.

The bars have a similar height (i.e. in the z direction) to the surrounding disk.

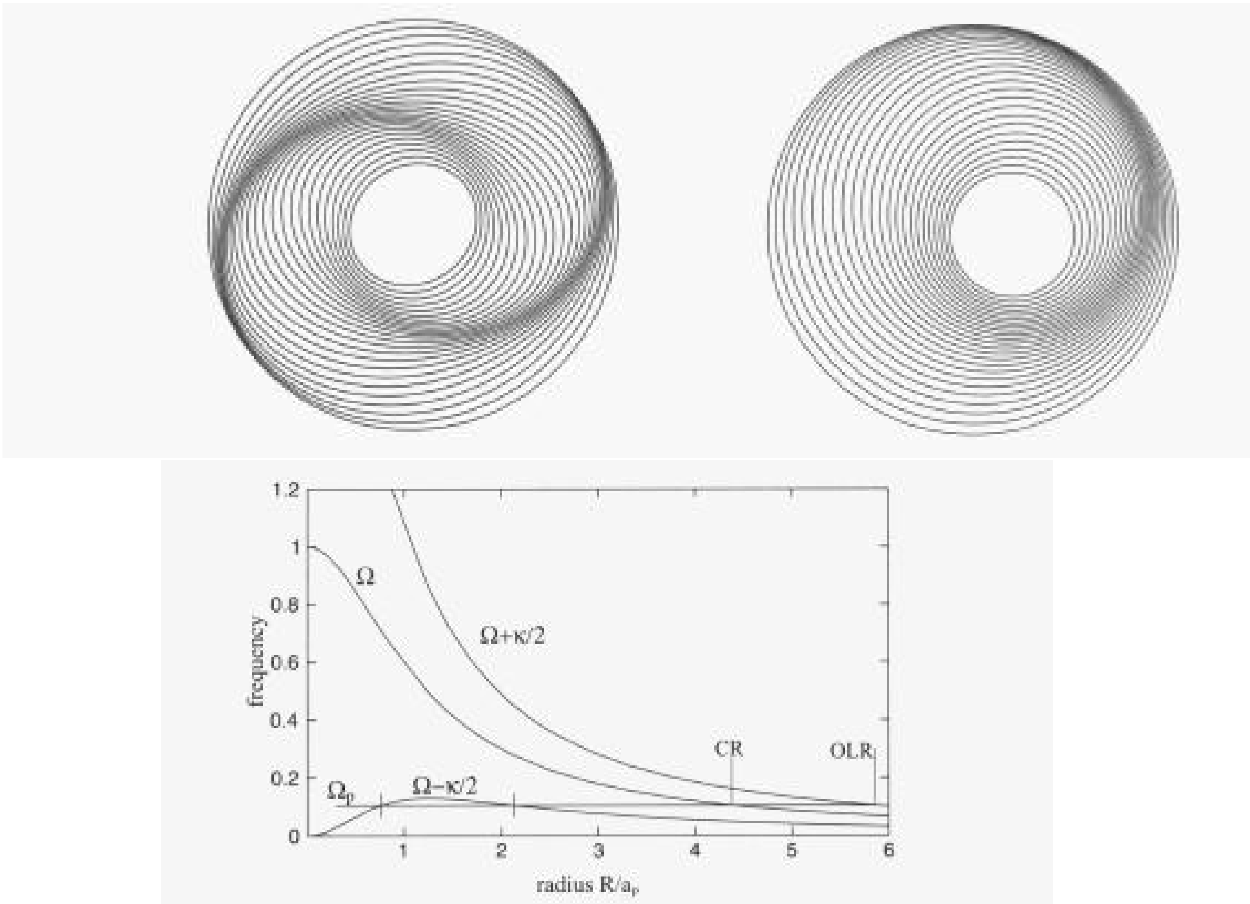

Figure 4.4: Top panels: a two armed and one-armed spiral pattern resulting nested ovals, which in turn result from epicyclic orbits. Bottom panel: Orbital frequencies from a Plummer potential. The 2 inner Lindblad resonances are marked with vertical ticks. The corotation radius is labelled "CR" and the outer Lindblad resonance is labelled "OLR".

The Milky War is believed to be a barred spiral.

The bar is not static (i.e. it rotates), but unlike spiral arms it is not a density wave – stars that are within bars tend to remain with the bar.

Stars and gas no longer follow near-circular paths but have a orbits that close on themselves (from the perspective of an observer rotating with the bar).

Spiral arms often appear to start from the end of bars – believed to be an illusion - bars and the disks should have different pattern speeds, with the bars having the higher pattern speeds.

Gas in a spiral galaxy can only flow inwards if it loses angular momentum – the asymmetric gravitational forces associated with a bar assist this.

Bars are good ways of funnelling gas towards the central black hole.

Bars often have regions of compressed dust/gas on their leading edges (see NGC1365 image).