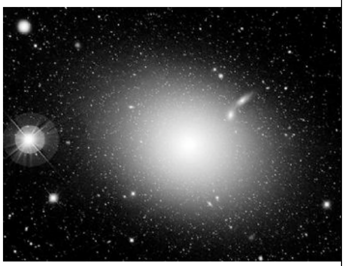
GALAXY FORMATION

Part VII

- The Jeans Mass
- Evolution of the Jeans Mass and the role of decoupling
- The spherical infall model
- Baryon cooling



How large a volume collpses to form a galaxy?

Consider a small ($\delta\rho/\rho$ <<1) spherical perturbation of radius r:

$$M(r) \cong (4\pi/3) r^3 \overline{\rho}_{matter} = (4\pi/3) r^3 (3H^2/8\pi G)\Omega_{matter}$$

For H_0=100h km/s/Mpc,

$$M(r)/M_{sun} \cong 1.16\times10^{12} h^2 \Omega_{matter} r^3_{Mpc}$$

For h=0.7 and Ω_{matter}=0.25,

$$M(r)/M_{sun} \cong 1.4\times10^{11} r^3_{Mpc}$$

i.e. a typical large galaxy coalesced from matter within a comoving volume of roughly ~ 1-1.5 Mpc radius

Jeans mass in an expanding Universe

- Baryonic matter will be held up from collapse by pressure until an overdense region larger that the Jeans length has been assembled
- The Jeans length depends on the sound speed and density of matter

$$2r \gtrsim \sqrt{\frac{15}{\pi}}\sqrt{\frac{c_s^2}{G\rho}} \approx \lambda_J, \text{ where } \lambda_J \equiv c_s\sqrt{\frac{\pi}{G\rho}}.$$

Early on, while the Universe is radiation-dominated, the density $\rho_r = a_B T^4/c^2$ is low and the pressure is high, with $c_s = c/\sqrt{3}$. So Equation 8.71 gives

$$\lambda_J = c^2\left(\frac{\pi}{3Ga_B T^4}\right)^{1/2} \propto T^{-2}. \tag{8.72}$$

The *Jeans mass* $\mathcal{M}_J$ is the amount of matter in a sphere of diameter λ_J:

$$\mathcal{M}_J \equiv \frac{\pi}{6}\lambda_J^3 \rho_m, \tag{8.73}$$

Jeans mass in an expanding Universe

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- The energy density of an ordinary sound wave is positive, However, the gravitational energy density of a sound wave is negative, since the enhanced attraction in the compressed regions overwhelms the reduced attraction in the dilated regions.
- The Jeans instability sets in when the net energy density becomes negative, so that the system can evolve to a lower energy state by allowing the wave to grow, and thus the system to fragment.

Jeans mass: radiation dominated

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Consider, for instance, the "radiation" component of the universe. With $w = 1/3$, the sound speed in a gas of photons or other relativistic particles is

$$c_s = c/\sqrt{3} \approx 0.58c \; . \tag{12.25}$$

The Jeans length for radiation in an expanding universe is then

$$\lambda_J = \frac{2\pi\sqrt{2}}{3} \frac{c}{H} \approx 3.0 \frac{c}{H} \; .$$

Jeans mass: matter dominated

(ii) After recombination, radiation pressure is negligible, so the sound speed

$$c_s^2 = dP/d\rho = \left(\frac{\gamma k_B T}{\mu m_H} \right)^{1/2},$$

where μ is the relative mass of each particle. Thus, the Jeans Mass

$$M_J = \frac{\pi}{6} \rho_m \left(\frac{\pi k_B T}{G \rho_m m_H} \right)^{3/2}.$$

Therefore $M_J \propto T^{3/2} \rho_m^{-1/2}$.

The Jeans mass in the radiation dominated era is of the order of $10^{16} M_\odot$ (scale of galaxy clusters). Adiabatic baryonic perturbations with scale sizes smaller than superclusters cannot grow before recombination.

After recombination, the Jeans mass abruptly falls to globular cluster masses ($\sim 10^5 M_\odot$). At this point, all scales of mass $> 10^5 M_\odot$ become unstable, and pertur-

Jeans mass before decoupling

if we regard the baryons as a minor contaminant, the Jeans length of the photon-baryon fluid was roughly the same as the Jeans length of a pure photon gas:

$$\lambda_J(\text{before}) \approx 3c/H(z_{\text{dec}}) \approx 0.6\,\text{Mpc} \approx 1.9 \times 10^{22}\,\text{m} \ . \tag{12.27}$$

The *baryonic Jeans mass*, M_J, is defined as the mass of baryons contained within a sphere of radius λ_J;

$$M_J \equiv \rho_{\text{bary}} \left(\frac{4\pi}{3} \lambda_J^3 \right) \ . \tag{12.28}$$

Immediately before decoupling, the baryonic Jeans mass was

$$\begin{aligned} M_J(\text{before}) &\approx 5.0 \times 10^{-19}\,\text{kg}\,\text{m}^{-3} \left(\frac{4\pi}{3} \right) (1.9 \times 10^{22}\,\text{m})^3 \\ &\approx 1.3 \times 10^{49}\,\text{kg} \approx 7 \times 10^{18}\,\text{M}_\odot \ . \end{aligned} \tag{12.29}$$

This is approximately 3×10^4 times greater than the estimated baryonic mass of the Coma cluster, and represents a mass greater than the baryonic mass of even the largest supercluster seen today.

Jeans mass after decoupling

Now consider what happens to the baryonic Jeans mass immediately after decoupling. Once the photons are decoupled, the photons and baryons form two separate gases, instead of a single photon-baryon fluid. The sound speed in the photon gas is

$$c_s(\text{photon}) = c/\sqrt{3} \approx 0.58c \ . \tag{12.30}$$

The sound speed in the baryonic gas, by contrast, is

$$c_s(\text{baryon}) = \left(\frac{kT}{mc^2} \right)^{1/2} c \ . \tag{12.31}$$

At the time of decoupling, the thermal energy per particle was $kT_{\text{dec}} \approx 0.26\,\text{eV}$, and the mean rest energy of the atoms in the baryonic gas was $mc^2 = 1.22 m_p c^2 \approx 1140\,\text{MeV}$, taking into account the helium mass fraction of $Y_p = 0.24$. Thus, the sound speed of the baryonic gas immediately after decoupling was

$$c_s(\text{baryon}) \approx \left(\frac{0.26\,\text{eV}}{1140 \times 10^6\,\text{eV}} \right)^{1/2} c \approx 1.5 \times 10^{-5} c \ , \tag{12.32}$$

only 5 kilometers per second. Thus, once the baryons were decoupled from the photons, their associated Jeans length decreased by a factor

$$F = \frac{c_s(\text{baryon})}{c_s(\text{photon})} \approx \frac{1.5 \times 10^{-5}}{0.58} \approx 2.6 \times 10^{-5} \ . \tag{12.33}$$

Decoupling causes the baryonic Jeans mass to decrease by a factor $F^3 \approx 1.8 \times 10^{-14}$, plummeting from $M_J(\text{before}) \approx 7 \times 10^{18}\,\text{M}_\odot$ to

$$M_J(\text{after}) = F^3 M_J(\text{before}) \approx 1 \times 10^5\,\text{M}_\odot \ . \tag{12.34}$$

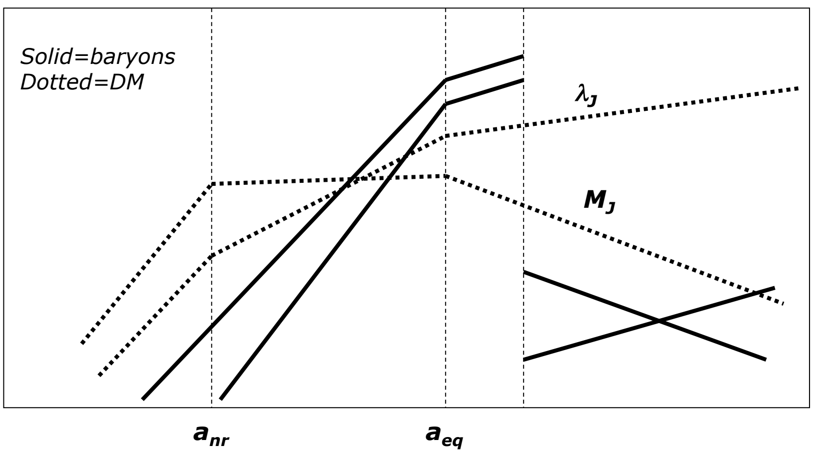
$$M_J(\text{after}) = F^3 M_J(\text{before}) \approx 1 \times 10^5\, \mathrm{M}_\odot\ . \qquad (12.34)$$

This is comparable to the baryonic mass of the smallest dwarf galaxies known, and is very much smaller than the baryonic mass of our own Galaxy, which is $\sim 10^{11}\, \mathrm{M}_\odot$.

The abrupt decrease of the baryonic Jeans mass at the time of decoupling marks an important epoch in the history of structure formation. Perturbations in the baryon density, from supercluster scales down the the size of the smallest dwarf galaxies, couldn't grow in amplitude until the time of photon decoupling, when the universe had reached the ripe old age of $t_{\text{dec}} \approx 0.35\,\text{Myr}$. After decoupling, the growth of density perturbations in the baryonic component was off and running. The baryonic Jeans mass, already small by cosmological standards at the time of decoupling, dropped still further with time as the universe expanded and the baryonic component cooled.

Gravitational Instability in an Expanding Universe - 10

Evolution of Baryonic Matter Perturbations



Nonlinear Regime: the Spherical Infall Model

The initial evolution of perturbations, when the fractional overdensity δ is small can be considered in the *linear regime.* However, when $\delta \sim 1$ the evolution becomes non-linear.

What happens next can be analysed using the ***Spherical Infall Model .***

To summarize (see notes for details and derivations):
- A slightly overdense region initially expands, but at a slightly lower rate than the universal rate
- The expansion of each shell of the overdense region eventually stops and the motion "turns around" initiating collapse
- The collapse is followed by a process of ***violent relaxation*** that reshuffles, randomizing ,the orbits of the infalling particles, leading to ***virialization.***
- Through dissipative processes, baryons lose energy and fall deeper in the potential well of DM.
- If the cooling time of the baryon gas is smaller than the collapse time, fragmentation will take place and smaller units can collapse

Nonlinear Regime: the Spherical Infall Model

A couple of important results of the spherical infall model analysis:

1. A perturbation of amplitude δ_i at redshift z_i will turn around at

$$1 + z_{turn} \cong 0.57(1 + z_i)\delta_i$$

E.g. an overdensity of 1% at z=1000 will turn around at z~4.7 at which time will be overdense by a factor of 4.6 wrt the background

2. The turn around overdensity is always 4.6 times the background density, independent on initial conditions

$$1 + \delta_{turn} = 9\pi^2 / 16 \cong 5.6$$

Nonlinear Regime: the Spherical Infall Model

We define a virialized system as one where ***E=U+K=-K***, i.e. **|*U*|=2*K***

At turn around time, all the energy is in ***U***, so that ***E=U~3GM²/5R****max*

We define ***virial velocity , virial radius:***

$$|U| = 3GM^2/5R_{vir} \qquad R_{vir} = R_{turn}/2 \qquad v_{vir} = (6GM/5R_{turn})^{1/2}$$

and the ***redshift of virialization***

$$1+z_{vir} = 0.36(1+z_i)\delta_i = 0.63(1+z_{turn})$$

… as for the density at the epoch of virialization:

$$\rho_{vir} \cong 2^3 \rho_{turn} \cong 8\times 5.6\times \rho_{bg}(z_{turn}) \cong 8\times 56\times \left(\frac{1+z_{turn}}{1+z_{vir}}\right)^3 \rho(z_{vir})$$

$$\boxed{\cong 180\rho_o(1+z_{vir})^3}$$

Nonlinear Regime: the Spherical Infall Model

The spherical infall model also yields scaling relations between size, mass and velocity which can be compared with observed relations.

$$R_{vir} = (164\ kpc)(\Omega_{matter}h^2)^{-1/3}(M/10^{12}M_{sun})^{1/3}(1+z_{vir})^{-1}$$

$$V_{vir} = (125\ km/s)(\Omega_{matter}h^2)^{1/6}(M/10^{12}M_{sun})^{1/3}(1+z_{vir})^{1/2}$$

We can also define a ***Virial Temperature*** via

$$(3/2)\mu T_{virial} = (1/2)V_{vir}^2$$

… yielding:

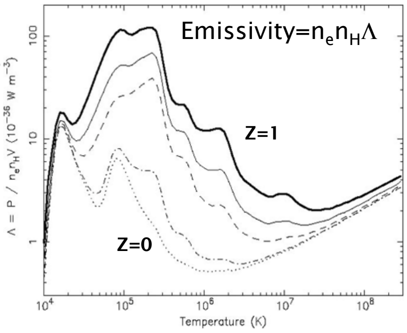
$$T_{vir} = (3.68\times 10^5 K)(\Omega_{matter}h^2)^{1/3}(M/10^{12}M_{sun})^{2/3}(1+z_{vir})$$

Cooling

The cooling function of a primordial baryonic fluid, i.e.

Λ(T) ➔ *energy loss by radiation p.u. time p.u. volume*

is a function of T and contributed to by 3 main processes:



1. Compton cooling with the CMB photons (important only at z>10)
2. Thermal bremsstrahlung (free-free) at T > 10^6 K
3. Bound-bound and recombination transitions of H and He at 10^4 and 10^5 K

[If elements > He are present, cooling rates below 10^6 K increase dramatically.
We define ***cooling time*** ➔

$$t_{cool} = \frac{E}{|dE/dt|} = \frac{3n_b kT}{n_b^2 \Lambda(T)}$$

which is useful to compare to the Gravitational collapse time

$$t_{dyn} \approx (G\rho)^{-1/2} \propto n_b$$

By setting $\boldsymbol{t_{cool} = t_{dyn}}$, we obtain a relationship between density and temperature. Systems hotter (i.e. more massive) than this will lead to hot hydrostatic halos of gas.