FEG PROBLEM SHEET 1

Some of these problems will be worked through in lectures, others can be addressed in the supervision classes or revision lectures.

1. A cluster of galaxies of radius 0.5 Mpc consists of 300 galaxies and has a one-dimensional velocity dispersion of 1000 km s^{-1}. Assuming that all the cluster mass resides in the galaxies, estimate both its crossing time and relaxation time. Compare these estimates to the age of the Universe and comment on the expected dynamical state of the cluster. How would your answer for relaxation time be changed if most of the cluster mass was *not* attached to the galaxies?

2. In a spherical galaxy, the circular velocity is given by

$$v_c^2 = \frac{ar^2}{(r^2+b^2)^{3/2}} ,$$

   where a and b are constants.

   (a) Derive the gravitational potential of this galaxy as a function of radius r.

   (b) Show that for large radii ($r \gg b$) the potential approaches that of a point mass.

   (c) Using Poisson's equation, find the density of the galaxy as a function of radius.

   (d) Using the substitution $r = b\tan\theta$, show that the mass of this galaxy is finite, and find its value.

3. A globular cluster, consisting of 2.3×10^5 identical stars of the same mass as the Sun, has a velocity dispersion $\sigma = 10$ km s^{-1}. Its angular radius on the sky is 3 arcmin. What is its distance from us?

4. Find the speed v_c of a star moving in a circular orbit about the centre of a singular isothermal sphere of velocity dispersion σ (independent of radius), where the radial distribution of mass is given by $M(r) = 2\sigma^2 r/G$.

5. Starting from the equations in your notes, show that for a Keplerian potential (i.e. central point mass) the epicyclic motion of orbiting stars has a tangential/radial axis ratio of 2, whilst a parabolic (harmonic oscillator) potential gives an axis ratio of 1.

6. A gas cloud of density ρ and sound speed c_s is collapsing under its own gravity. If the dispersion relation for acoustic waves is $\omega^2 = k^2c_s^2 - 4\pi G\rho$, where $k \equiv 2\pi/\lambda$ is the wavenumber, derive the Jeans length of the cloud in terms of the given quantities. What is the physical signicance of the Jeans length in the context of the evolution of the cloud?