FEG PROBLEM SHEET 2

Some of these problems will be worked through in lectures, others can be addressed in the supervision classes or revision lectures.

1. The observed *Fundamental Plane* relationship between the effective radius, velocity dispersion and mean surface brightness of elliptical galaxies is

   $$R_e \propto \sigma^{1.24} I^{-0.82}.$$

   One way of understanding this relation is to appeal to the virial theorem, but to allow for the possibility that the mass-to-light ratio (M/L) of galaxies varies systematically from galaxy to galaxy. What dependence of M/L on σ and I is required to account for the above *Fundamental Plane* relation?

2. Bremsstrahlung dominates the cooling of gas with temperatures higher than $\sim 10^7$ K. If the emissivity due to bremsstrahlung is proportional to $\sqrt{T}$, show how the cooling time of gas increases with temperature in this regime.

3. The nearby Seyfert galaxy NGC 4258 contains fast-moving water masers close to its nucleus. If these are observed to have line-of-sight velocities of 1000 km s^{-1}, and lie at a projected distance of 0.2 pc from the nucleus, then calculate an approximate value for the mass of the central black hole. What are the major uncertainties in your estimate? Calculate also the Schwarzschild radius of the central mass, and compare this with the distance of the masers from the centre.

4. Show that a black hole accreting matter at a rate limited by the Eddington luminosity grows in mass exponentially, and calculate the e-folding time. How long would a 10 $M_\odot$ black hole take to grow into a 10^9 $M_\odot$ in this way? Comment on the implications of your answer.

5. Assuming that the Milky Way contains $5 \times 10^9 M_\odot$ of cool ($T \sim 100$ K) atomic hydrogen, and that this is uniformly distributed within a disc of thickness 100 pc and radius 5 kpc, show that this substantially exceeds the Jeans Mass. If this gas were to collapse on a free-fall timescale, what would be the resulting star formation rate within the galaxy?

6. Show that equation 10.8 in your notes follows from the basic metallicity evolution equation (eq. 10.6) for the closed box model.

7. In the solar neighbourhood, the mean metallicity of the gas at the present time is observed to be $0.7Z_\odot$. The fact that only 2% of nearby stars are observed to have $Z < 0.25Z_\odot$, could be explained within the closed box model if the gas were pre-enriched before star formation started in the solar vicinity. If the yield is $p = 0.5Z_\odot$ then what level (Z_0) of pre-enrichment is required to make this work?