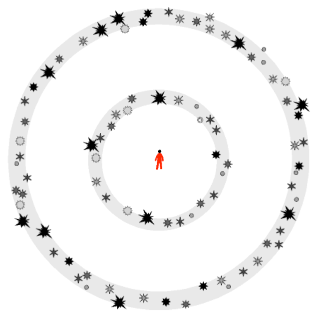
Lecture 14: Cosmology

- Olbers' paradox
- Redshift and the expansion of the Universe
- The Cosmological Principle
- Ω_0 and the curvature of space
- The Big Bang model
  - Primordial nucleosynthesis
  - The Cosmic Microwave Background
- The age and future of the Universe

Olbers' Paradox

- Named for Wilhelm Olbers, but known to Kepler and Halley
  - Consider spherical shell of radius r and thickness dr
  - Number of stars in this shell is $4\pi r^2 n$ dr, where n is number density of stars
  - Flux from each star is $L/4\pi r^2$, therefore flux from shell is $F = 4\pi r^2 n \, dr \, L/4\pi r^2 = nL \, dr$, independent of r
  - therefore, in an infinite universe, night sky should be as bright as a typical stellar surface – since any line of sight will hit a star, and stars block light from behind them
- So - why is the sky dark at night?

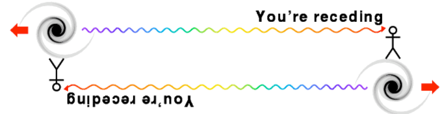


Resolutions of Olbers' paradox

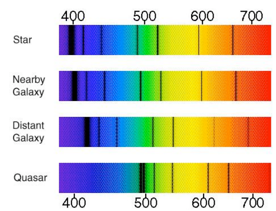
- Light is absorbed by intervening dust
  - suggested by Olbers
  - doesn't work: dust would heat up over time until it reached the same temperature as the stars that illuminate it
- Universe has finite size
  - suggested by Kepler
  - this works (integral is truncated at finite r)
  - but static finite universe would collapse under its own gravity - i.e. unstable
- Universe has finite age
  - light from stars more than ct_{Uni} distant has not had time to reach us
- Universe is expanding
  - effective temperature of distant starlight is redshifted down
  - this effect not known until 19th century
- These last two provide the resolution to Olbers' paradox

Cosmic redshift

- When a galaxy is receding, light waves travelling to us are redshifted



- Hubble measured the spectrum of nearby galaxies and found the spectral lines to be redshifted
- The more distant the galaxy, the greater its redshift



Cosmic redshift

Redshift z is given by:

$$1 + z = \frac{\lambda}{\lambda_0} \quad \text{or} \quad z = \frac{\Delta\lambda}{\lambda_0}$$

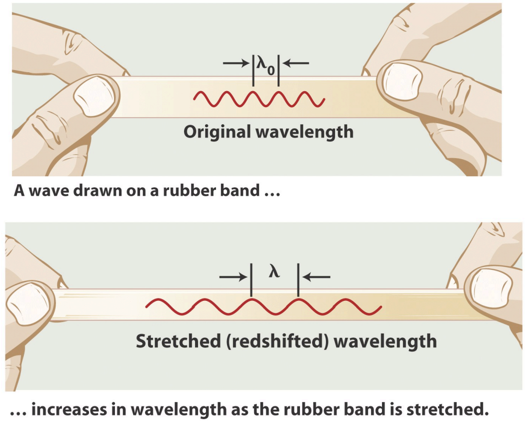
Where λ is the observed wavelength, λ_0 is the rest-frame wavelength and $\Delta\lambda = \lambda - \lambda_0$

For small z (i.e. $v \ll c$):

$$z = \frac{v}{c}$$

Where v is the recession velocity and c is the speed of light.

Cosmic redshift



It is more useful to think of the cosmic redshift as arising from the expansion of space, rather than being a Doppler shift due to the recession of the galaxies.

The Cosmological Principle

The Cosmological Principle:

Cosmological theories are based on the idea that on sufficiently large scales, the Universe looks the same

•at all locations (homogeneity),

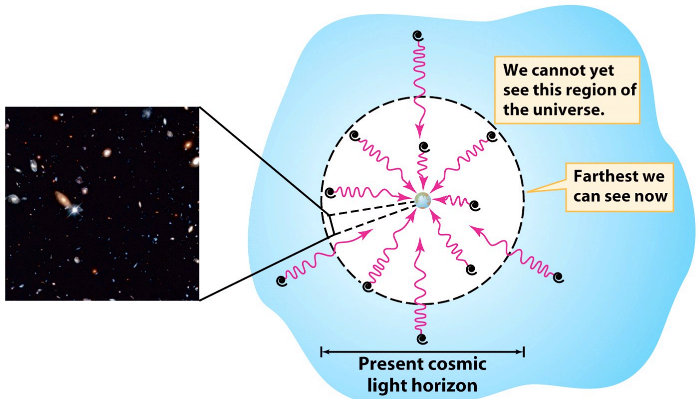
and

•in every direction (isotropy).

One implication of this is that the Universe cannot have an "edge".

Note that this is an assumption.

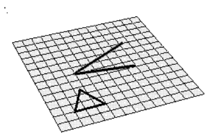
The observable universe



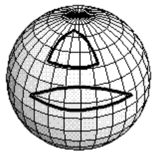
We cannot see objects beyond a light travel time of ~14 Gyr, because light from them has not had enough time to reach us since the beginning of the Universe.

The curvature of space

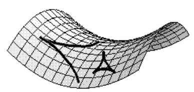
- According to General Relativity - space is warped by mass
- Cosmological space is also being stretched by the expansion
- The net curvature of space in the Universe depends upon the balance between these two effects
- 3 simple types of curvature - consider 2-D analogies



Flat - no curvature



Positive curvature - e.g. sphere



Negative curvature - e.g. saddle

The Density Parameter - Ω_0

Ω_0 is defined as:

$$\Omega_0 = \frac{\rho_0}{\rho_c}$$

Where ρ_0 is the current density of the universe and ρ_c is the *critical density* – the density which makes space flat.

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

So:

- $\Omega_0 < 1$ gives negative curvature – **Open Universe**
- $\Omega_0 = 1$ gives a **Flat Universe**
- $\Omega_0 > 1$ gives positive curvature - a finite, **Closed Universe**

The Hot Big Bang model

If the universe is expanding, then it follows that by going back in time, we move towards a moment when the universe was infinitely dense.

This *singularity*, from which the universe is expanding is the so-called **Big Bang**.

The Big Bang model is a theory for the formation, development and future history of the universe. It is now widely accepted because it explains three very important observational facts:

1. The expansion of the Universe
2. The relative abundances of light elements
3. The cosmic microwave background

Primordial Nucleosynthesis

For the first 10-15 minutes after the Big Bang, the temperature of the Universe was very high – high enough to produce light elements from hydrogen by nuclear fusion.

The theory of **primordial nucleosynthesis** combines our understanding of nuclear physics with the Big Bang model, to predict the relative abundances of the light elements:

Deuterium (^2_1H), Helium-3 (^3_2He), Helium-4 (^4_2He), Lithium-7 (^7_3Li)

These predictions can be tested by studying metal-poor galaxies (i.e. those where stellar-processing has had very little effect)

Predictions match observations very well.

Cosmic Microwave Background

In the early stages of the Universe, the temperature was high enough to keep all matter fully ionized.

Photons scattered on all the free electrons, keeping matter and radiation in *equilibrium.*

However, once the temperature of the Universe dropped to ~3000K, neutral hydrogen could form – this was about ½ million years after the Big Bang.

At this point, the number of free electrons dropped dramatically, and the photons became free to move independently of matter:

Era of decoupling (or recombination)

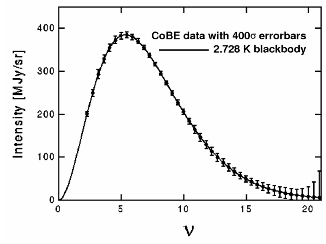
Cosmic Microwave Background

So, at decoupling the Universe was filled with radiation with blackbody temperature T~3000K.

Since then expansion has caused the radiation to "cool".

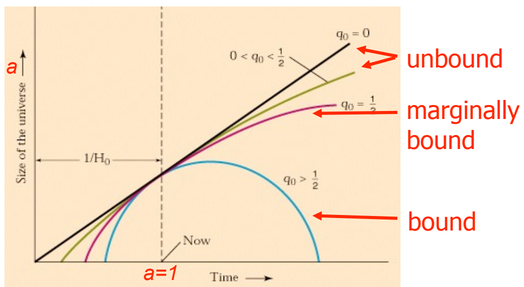
Big Bang theory, therefore, predicts a "sea" of photons filling the Universe now with a blackbody temperature of ~3K.

In 1965 Penzias and Wilson accidentally discovered the radiation using a microwave detector (a Nobel prize was to follow).



The spectrum of the CMB is extremely close to that of a blackbody with T=2.726 +/- 0.010 K

The scale factor of the Universe



From the cosmological principle, the expansion of the Universe must be the same everywhere. It can therefore be represented by the changing value of a scale factor *a*, which is normally defined to be 1 at the present time. For an empty Universe, *da/dt*=const., whilst matter decelerates *a*.

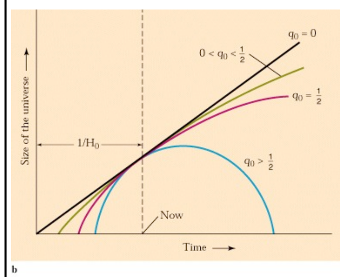
The Hubble parameter *H* is then just *H=(da/dt)/a*

H_0 ("Hubble constant") is the current value of the Hubble parameter.

The Age of the Universe

For a massless Universe, the expansion rate da/dt is a constant, so that the time since the Big Bang is

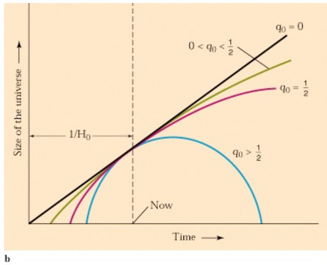
$$t_0 = \frac{1}{H_0} = \frac{1}{70km/s/Mpc} = \frac{1s.1Mpc}{70km}\frac{10^6\,pc}{1Mpc}\frac{3\times10^{18}\,km}{1pc}\frac{1yr}{3\times10^7\,s} = 14 \text{ Billion years}$$



$1/H_0$ is known as the Hubble time.

So the age of the Universe is equal to the Hubble time if it has expanded at a constant rate. Otherwise the Hubble time gives just a rough measure of the age of the Universe.

The Age of the Universe



$$\Omega = \frac{\rho}{\rho_{crit}}$$

In general the expansion of the Universe could be either decelerating or accelerating.

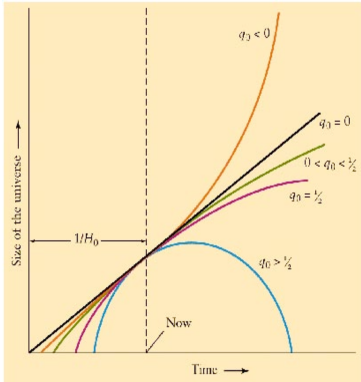
The rate of deceleration of the expansion can be represented by a deceleration parameter, q_0 .

A massive Universe with Ω_0=1 has q_0=1/2 , and an age

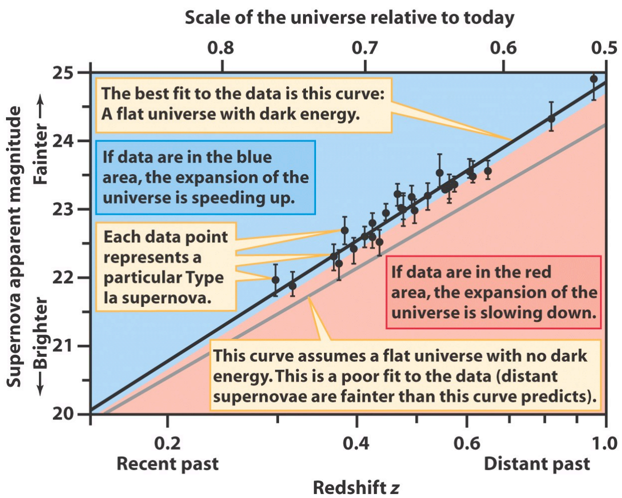
$$t_0 = \frac{2}{3}\frac{1}{H_0} = \frac{2}{3}\frac{1}{70 km/s/Mpc} = 9.3 \text{ Billion years}$$

For comparison, the oldest globular clusters in the Galaxy are about 13 Gyr old…

The future of the Universe



- There is good evidence (e.g. from the size scales of fluctuations in the CMB) that Ω_0=1.
- However, observations of high redshift supernovae now show that the expansion of the Universe seems to be accelerating (i.e. q_0 is -ve).
- This is believed to be due to dark energy (also known as Einstein's cosmological constant), which now dominates the energy density of the Universe.
- If this is the case, then the future of the Universe will be one of ever-accelerating expansion.



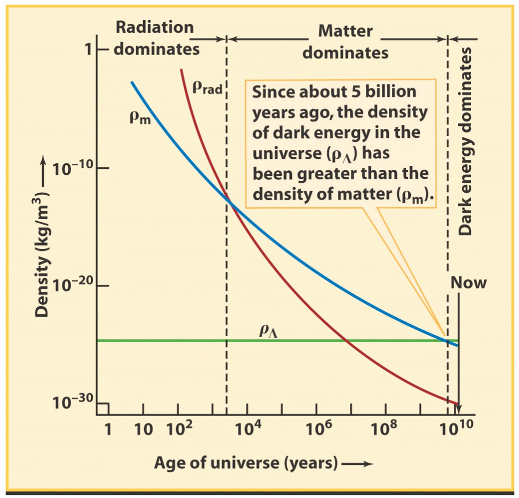


table 28-2 Some Key Properties of the Universe

Quantity	Significance	Value*
Hubble constant, H_0	Present-day expansion rate of the universe	71^{+4}_{-3} km/s/Mpc
Density parameter, Ω_0	Combined mass density of all forms of matter *and* energy in the universe, divided by the critical density	1.02 ± 0.02
Matter density parameter, Ω_m	Combined mass density of all forms of matter in the universe, divided by the critical density	0.27 ± 0.04
Density parameter for ordinary matter, Ω_b	Mass density of ordinary atomic matter in the universe, divided by the critical density	0.044 ± 0.004
Dark energy density parameter, Ω_Λ	Mass density of dark energy in the universe, divided by the critical density	0.73 ± 0.04
Age of the universe, T_0	Elapsed time from the Big Bang to the present day	$(1.37 \pm 0.02) \times 10^{10}$ years
Age of the universe at the time of recombination	Elapsed time from the Big Bang to when the universe became transparent, releasing the cosmic background radiation	$(3.79^{+0.08}_{-0.07}) \times 10^5$ years
Redshift z at the time of recombination	Since the cosmic background radiation was released, the universe has expanded by a factor $1 + z$	1089 ± 1

**Values are from the first year of WMAP data. (NASA/WMAP Science Team)*