Lecture 3: Luminosity, brightness and telescopes

- Luminosity and the Stefan-Boltzmann law
- Solid angle, flux, brightness and intensity
- Astronomical magnitudes - apparent and absolute
- Telescopes and limiting resolution

Luminosity and the Stefan-Boltzmann law

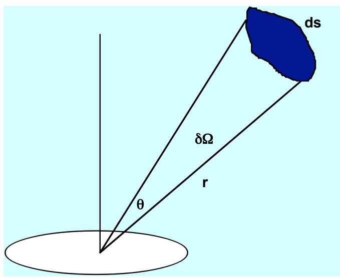
- The luminosity of a body is the total power radiated by it - measured in Watts in SI units, but often in erg s^{-1} (1 W=10^7 erg s^{-1}) or in units of the solar luminosity ($L_\odot$=3.9x10^{26} W) in the astronomical literature.
- For a blackbody radiator (a reasonable approximation for stars) the flux of energy emitted from the surface (in W m^{-2}) is given by the Stefan-Boltzmann law: $F=\sigma T^4$, where the Stefan-Boltzmann constant is σ=5.67x10^{-8} W m^{-2} K^{-4}.
- Hence the total luminosity of a star is $L=4\pi R^2\ \sigma T^4$.
- Example: for the sun T≈ 6000 K, $R_\odot$= 6.96x10^8 m, so that $L_\odot$≈4x10^{26} W.

Solid Angle

The solid angle subtended by an object at an observer is

$$d\Omega=ds/r^2,$$

where *ds* is the area of the object perpendicular to the line of sight, and *r* is its distance.



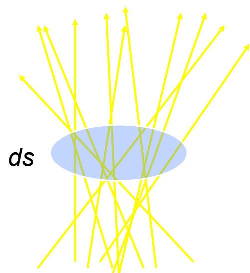
Solid angle is therefore dimensionless, and is usually measured in steradians (sr)
1 sr = 1 rad^2 = $(57.3)^2$ sq. deg.

The whole sky subtends an angle of 4π steradians.

Flux, brightness and intensity

The flux (F) through a surface is the total power per unit area flowing through it (in W m^{-2}). In *Universe*, this is mostly called apparent brightness. The flux through a sphere at distance *d* from a source of luminosity *L* is

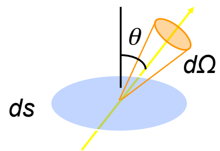
$$F = L/4\pi d^2$$



The intensity (I) is the flux per unit solid angle in some particular direction, so that

$$dE = I \cos\theta\ ds\ d\Omega\ dt$$

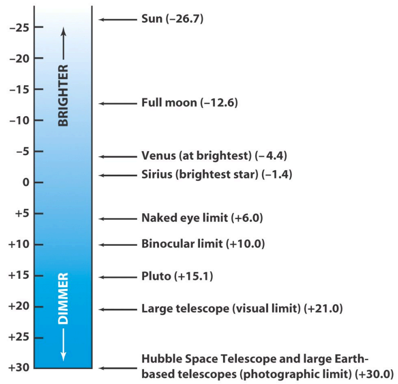
is the energy flowing through the element of area *ds* into solid angle element *d*Ω, as shown.



Intensity (and similarly flux) can also be considered as a function of frequency, this is known as specific intensity, or spectral intensity, and denoted I_ν, which has units of W m^{-2} Hz^{-1} sr^{-1}.

Astronomers often use the magnitude scale to denote brightness

- Historically, the apparent magnitude scale for stars ran from 1 (*brightest*) to 6 (*dimmest*).
- Today, this scale extends into the negative numbers for really bright objects, and into the 20s and 30s for really dim ones.
- Absolute magnitude is how bright an object would look if it were 10 pc away.



Some apparent magnitudes

Apparent magnitude

Magnitude is a logarithmic scale, defined such that 2.5 magnitudes correspond to a change in brightness (i.e. flux) by a factor of 10.

i.e. $m_1 - m_2 = -2.5 \log (F_1/F_2)$

Q: Why the minus sign on the RHS?

Conversely, the flux ratio can be derived from the apparent magnitudes via

$$F_1/F_2 = 10^{-(m_1-m_2)/2.5}$$

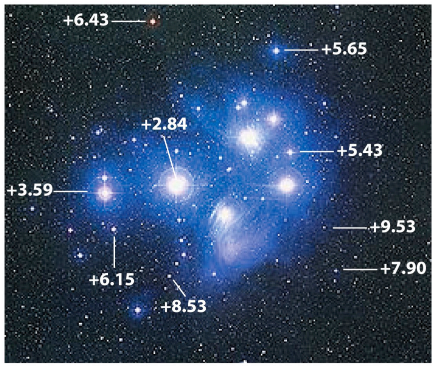
For a source of given luminosity, how does the apparent magnitude depend upon its distance?

Flux falls off as distance squared, so for two objects of the same L but distances d_1 and d_2, the flux ratio is

$$F_1/F_2 = (d_2/d_1)^2,$$

and the magnitude difference is therefore (from the first equation above) $m_1 - m_2 = 5 \log(d_1/d_2)$.

How much brighter is the most luminous star in the Pleiades, than the faintest star marked in the figure below?



Apparent magnitudes of stars in the Pleiades

Answer:

Absolute magnitude M

The absolute magnitude of a star is the magnitude it would have if it were placed at d=10 pc. Hence, substituting for M in the magnitude-distance formula:

$$m - M = 5\log_{10}\left(\frac{d}{10\,pc}\right)$$

m-M is known as the distance modulus of the star.

E.g. for the sun (in the V band) m=-26.74 and d=1AU, so that

$$M = m - 5\log\frac{d}{10} = -26.74 + 31.57 = +4.83$$

Absolute magnitude is a measure of the luminosity of a star. This can be expressed in units of the solar luminosity.

E.g. Arcturus has M_V=-0.31, what is its luminosity?

Ans:

Absolute magnitudes - some examples

Star	Apparent magnitude	Distance (parsecs)	Absolute magnitude	Luminosity (relative to Sun)
Sun	-26.8		4.8	
Full Moon	-12.6			
Venus	-4.4			
Sirius	-1.44	2.64	1.45	22.5
Arcturus	-0.05	11.25	-0.31	114
Vega	0.03	7.76	0.58	50.1
Spica	0.98	80.40	-3.55	2250
Barnard's Star	9.54	1.82	13.24	1/2310
Proxima Centauri	11.01	1.30	15.45	1/17700

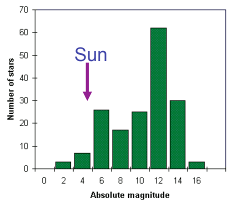
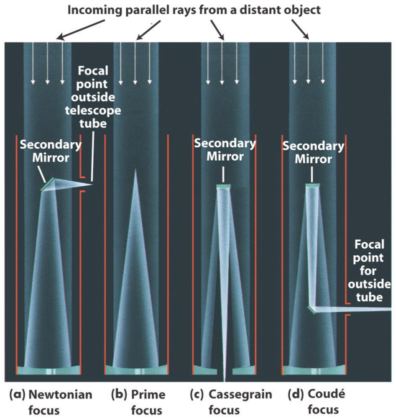
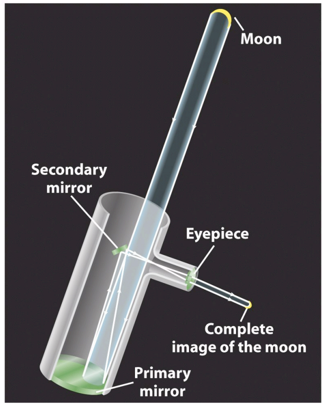


Figure 2. The frequency distribution of the absolute magnitudes of all stars within 10 parsecs of the Sun (from the Hipparcos database).

Reflecting Telescopes





Telescope images - limiting factors

A telescope's angular resolution, which indicates ability to see fine details, is limited by two key factors:

•Diffraction is an intrinsic property of light waves. According to the *Rayleigh criterion*, the diffraction-limited angular resolution for radiation of wavelength λ, using an imager of aperture diameter *D*, is

$$\theta = 1.22\ \lambda/D \text{ radians } (\approx 2.5\times10^{5}\ \lambda/D \text{ arcsec})$$

– Its effects can be reduced by using a larger objective lens or mirror (i.e. increasing *D*).

– For a 1m telescope imaging radiation with λ=500 nm, the diffraction limit is therefore ≈0.1″.

•The blurring effects of atmospheric turbulence can be minimized by placing the telescope atop a tall mountain with very smooth air. Even at very good sites, seeing is rarely much better than 0.5″.

– Atmospheric blurring can be dramatically reduced by the use of adaptive optics, and eliminated entirely by placing the telescope in orbit.