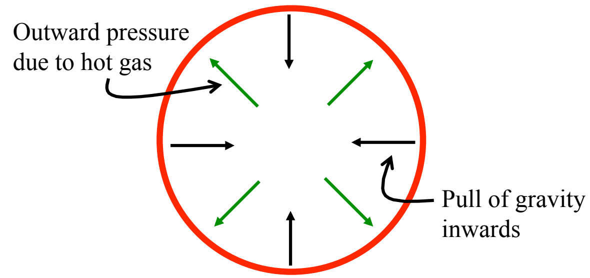
Lecture 7: The basic physical properties of a star

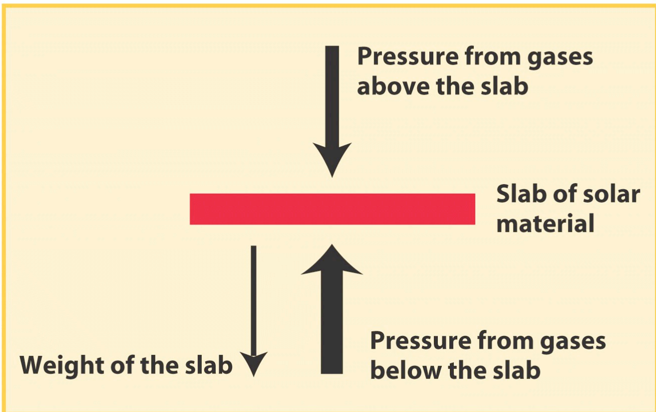
- Hydrostatic equilibrium and dynamical collapse time
- Global stability, mean temperature and the Virial Theorem
- Energy generation in stars
- Energy transport

Hydrostatic equilibrium

- A star is mostly made of hydrogen and helium
- It would collapse under its own gravity were it not for support from internal pressure
- The balance between gravity and an internal pressure gradient is known as hydrostatic equilibrium



Hydrostatic equilibrium requires a pressure *gradient*



Hydrostatic equilibrium

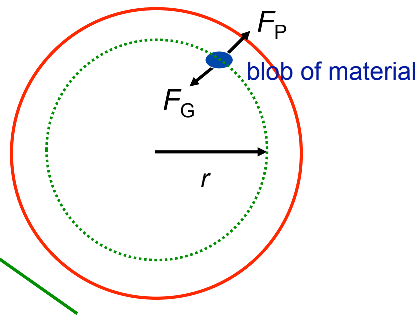
For a stable star, hydrostatic equilibrium must hold throughout the interior.

Consider a blob of material at radius r within a star:

- Outward pressure force F_P must balance inward gravitational force F_G
- Equating the force per unit volume due to pressure gradient and gravity gives

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$$

where P and ρ are the gas pressure and density at radius r, and $M(r)$ is the mass within this radius.



This is the equation of hydrostatic equilibrium

Dynamical collapse time

magine that the pressure support were suddenly to disappear. How long would it take a star to collapse?

Applying Newton's second law to an element of mass *m* at the stellar surface (radius *R*), gives

$$F_G = m\, dv/dt\,,$$

or $$GMm/R^2 = m\, d^2R/dt^2$$

We can set $d^2R/dt^2 \sim R/t_{dyn}^2$, where t_{dyn} is the dynamical collapse time - a rough measure of the timescale on which the system would collapse. From the equations above

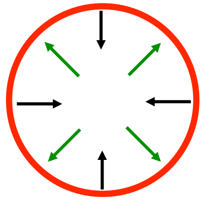
$$t_{dyn} = (R^3/GM)^{1/2}$$

Substituting values for the Sun (R=7x10^8 m, M=2x10^{30} kg), gives the startling result….

The Virial Theorem

- Hydrostatic equilibrium applies at every point within a static star.
- However, it is also possible to integrate over the whole star, to derive a *global* relationship between gravity and pressure.

$$3\int P dV = -\Omega = \eta \frac{GM^2}{R}$$ **V.T.**



where Ω is the total gravitational potential energy of the star (-ve), and η is a number of order unity, which depends on its detailed density profile.

Defining a mean temperature <*T*> for a star, and assuming *P=nkT:* $3\int P dV = 3k <T> \int n dV = 3Nk <T>$

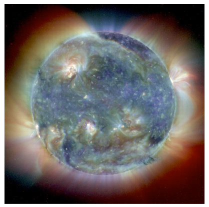
and therefore $$<T> = \frac{\eta G M \mu}{3kR}$$ ← This will prove very useful

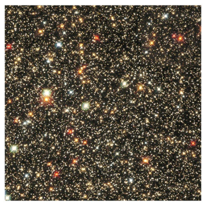
where μ is the mean mass per particle, $\mu = M/N$.

What is a star?

- The inside of a star must be hot, so that pressure can prevent gravitational collapse
- However, a star is constantly radiating energy, which must be replenished by some energy source
- The conflict between gravity and pressure determines the course of stellar evolution
- One very important consequence of the virial theorem, is that a star satisfying the equation $<T> = \frac{\eta G M \mu}{3kR}$ cannot cool.

  As it cools, it loses pressure support, and shrinks. As *R* decreases it must get *hotter*!





The Sun's energy supply

- Sun's luminosity = 3.9 x 10^{26} Watts

  What is the source of this energy?

- Age of sun is 4.5 Gyr, so total energy radiated to date is

  E_{tot}~6x10^{43} J

- Thermal energy?

  Total energy available = 3Mk<T>/2μ = 5x10^{34} T Joules.
  - So the mean temperature would have to have cooled from T>10^9 K to provide enough energy.
  - Not plausible.

The Sun's energy supply

- Gravitational potential energy? $\Omega = -\frac{3}{5}\frac{GM^2}{R} = -2 \times 10^{41}\ J$
  (uniform density sphere)

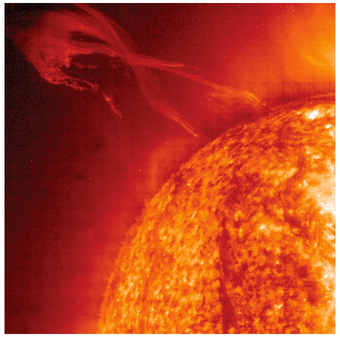
  With this energy, the sun would last $\frac{2\times10^{41}\,J}{3.9\times10^{26}\,J/s} = 1.7\times10^{7}\,yr$

- Nuclear energy? $4m_H - m_{He} = 0.029 m_H = 6\times10^{14}\,J/kg$

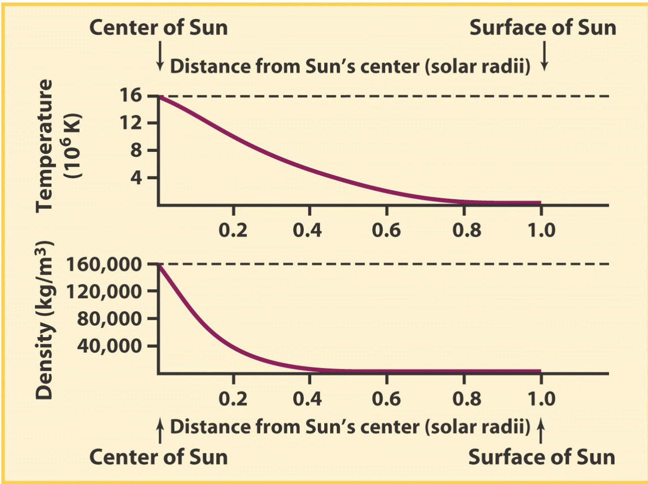
  $E = m\,c^2$

- Mass of sun= 2×10^{30} kg, so total energy available is $6\times10^{14} \times 2\times10^{30} \sim 10^{45}$ J. Compared to the $E_{tot} \sim 6\times10^{43}$ J radiated by the sun over its lifetime to date.
- Promising!

Thermonuclear fusion

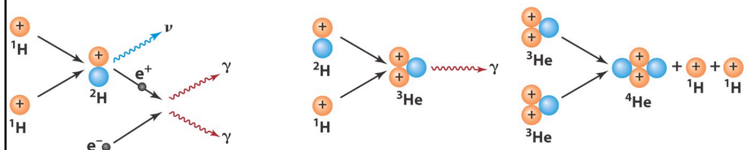


- The energy released in a nuclear reaction corresponds to a slight reduction of mass, according to Einstein's equation $E = mc^2$
- Thermonuclear fusion occurs only at very high temperatures; e.g. hydrogen fusion occurs only at temperatures in excess of about 10^7 K
- In the Sun, fusion occurs only in the dense, hot core
- It converts the most abundant element hydrogen, into helium



Hydrogen fusion occurs via a sequence of thermonuclear reactions with the net effect 4H→He

The proton-proton (or pp) chain

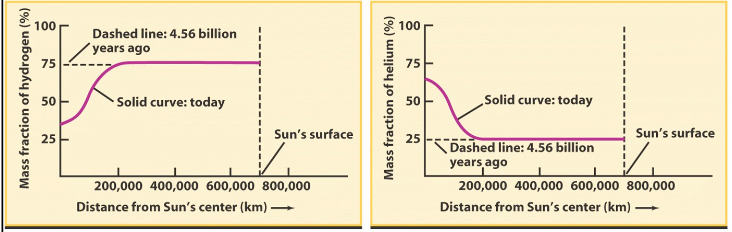


(a) Step 1:
- Two protons (hydrogen nuclei, ^{1}H) collide.
- One of the protons changes into a neutron (shown in blue), a neutral, nearly massless neutrino (ν), and a positively charged electron, or positron (e^+).
- The proton and neutron form a hydrogen isotope (^{2}H).
- The positron encounters an ordinary electron (e^-), annihilating both particles and converting them into gamma-ray photons (γ).

(b) Step 2:
- The ^{2}H nucleus from the first step collides with a third proton.
- A helium isotope (^{3}He) is formed and another gamma-ray photon is released.

(c) Step 3:
- Two ^{3}He nuclei collide.
- A different helium isotope with two protons and two neutrons (^{4}He) is formed and two protons are released.

Fusion is depleting H in the solar core



(a) Hydrogen in the Sun's interior (b) Helium in the Sun's interior

- The Sun has been a main-sequence star for about 4.56 Gyr
- It should remain one for about another 7 Gyr, at which point it will run out of hydrogen fuel in its core

Energy transport in the Sun

- Hydrogen fusion takes place in a core extending from the Sun's centre to about 0.25 solar radii
- The core is surrounded by a radiative zone extending to about 0.7 solar radii
  - In this zone, energy travels outward through radiative diffusion
- The radiative zone is surrounded by a rather opaque convective zone of gas at lower temperature and pressure
  - In this zone, energy travels outward primarily through convection