Year 1 Introduction to Astrophysics
Practice Problems 2 – Solutions

1. The virial theorem gives the expression

$$< T >= \frac{\eta G M \mu}{3kR}$$

relating the mean internal temperature of a star to its mass and radius. Hence, if such a star does not replace its radiated energy by an internal energy source, then it will lose pressure support and must shrink. However, the equation above tells us that in this case its internal temperature must <u>rise</u>.

The equation only applies provided the star is made of perfect gas (i.e. $P = nkT$). In the later evolutionary stages of low mass stars, the interior is supported by electron degeneracy pressure, and so the argument above breaks down, and the star *can* cool.

2. The M.S. lifetime is

$$t_{\rm MS} \approx 12 \left(\frac{M}{M_\odot}\right)^{-2.5} \ \text{Gyr},$$

which gives a M.S. a turnoff age of about 10^7 years, for a $15M_\odot$ star.

Since this timescale is short compared to the timescales of galaxies (e.g. one galactic rotation at the Sun's radius takes $\sim 2 \times 10^8$ years), this means that massive stars (and hence their surrounding HII regions) are found in locations where star formation is still active.

3. White dwarfs have much smaller radii than M.S. stars, since they are composed of degenerate matter. Since their radii are $\sim 0.01R_\odot$, their luminosity (which scales as R^2, from the blackbody formula) will be $\sim 10^4$ times smaller (i.e. 10 magnitudes fainter), at a given temperature.

4. A star will spin itself apart when the centrifugal force on its equator becomes as large as the gravitational force holding it together. This gives

$$mv^2/R = 4\pi^2 mR/P^2 = GMm/R^2.$$

Rearranging gives an expression for the period at which the star would disrupt:

$$P = 2\pi \left(\frac{R^3}{GM}\right)^{1/2}.$$

Taking $M = M_\odot$ and $R = 0.01R_\odot$, yields $P = 10\,\text{s}$. This is substantially longer that pulsar pulse periods, so pulsars cannot be white dwarfs.

5. The main lines of evidence for dark matter include: (i) flat rotation curves in spiral galaxies, and (ii) the high mass/light ratios of galaxy clusters (masses deduced from gravitational lensing, the virial theorem, or X-ray studies of hot gas).

6. If the Universe had expanded at a constant rate ($da/dt =$ const.) then its age would simply be $t_0 = a/\dot{a} = 1/H_0$. For $H_0 = 70$ km s^{-1} Mpc^{-1}, this gives $t_0 \approx 14$ Gyr.

   The gravitational attraction of matter causes the expansion of the Universe to slow down, so the simplest universe which would expand at constant velocity would be an empty universe! This would have $\Omega = 0$. [N.B. A low density universe ($\Omega \ll 1$) would slow down only gradually, and so would expand at an *almost* constant rate.]

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