Observational Cosmology – Unit 2 Solutions

5. The R-W metric is

$$ds^2 = dt^2 - \frac{a(t)^2}{c^2}\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2\right],$$

The interval ds is the proper time (you can see this because the proper time between two events is dt when the two events are measured to occur at the same point in space - ie. $dr = 0, d\theta = 0, d\phi = 0$).

Likewise the proper distance is $c\,ds$ for $dt = 0$ (two events measured to occur at the same time). Thus the proper distance between P and Q is

$$D_P = \int_P^Q c\,ds \qquad \text{for } dt = 0 \text{ and } a(t) = 1.$$

Since $d\theta = d\phi = 0$, we have

$$D_P = \int_0^r \frac{dr}{\sqrt{1-kr^2}}.$$

Since $k > 0$, this is clearly *greater* than r.