

## Observational Cosmology – Unit 5 Solutions

**2.** From the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P/c^2),$$

it follows that

$$q = -\frac{\ddot{a}}{aH^2} = \frac{4\pi G}{3H^2}(\rho_m + \rho_r + \rho_v + 3(P_m + P_r + P_v)/c^2).$$

Remembering that

$$\rho_c = \frac{3H^2}{8\pi G}, \quad P_M = 0, \quad P_r = \rho_r c^2/3, \quad \text{and} \quad P_v = -\rho_v c^2,$$

we obtain

$$q = \frac{1}{2\rho_c}(\rho_m + 2\rho_r - 2\rho_v) = \Omega_m/2 + \Omega_r + \Omega_\Lambda.$$