Observational Cosmology – Unit 5 Solutions

6. The angular size of an object of proper diameter l is $\theta = l/D_A$. Since the angular-diameter distance is $D_A = r/(1+z)$, combining this with the low redshift approximation

$$r \approx \frac{c}{H_0}[z - \frac{1}{2}(1+q_0)z^2]$$

gives

$$D_A \approx \frac{c}{H_0(1+z)}[z - \frac{1}{2}(1+q_0)z^2]$$

and therefore

$$\theta \approx \frac{H_0 l(1+z)}{cz[1-\frac{1}{2}(1+q_0)z]} \approx \frac{H_0 l}{cz}\left[1 + \frac{z(3+q_0)}{2}\right].$$

In a matter-dominated universe, this equation clearly predicts that θ increases with q_0. The physical reason for this can be appreciated by considering the angular size subtended by an object on the surface of a sphere. The smaller the radius of curvature of the sphere, the larger will be the angle subtended by a bar of given length at a certain proper distance from the observer.