Observational Cosmology - Assessed Exercise 2

1. Suppose that we live in a matter-dominated Universe with $\Lambda = 0$, $H_0 = 70$ km s^{-1} Mpc^{-1} and current matter density $\rho_0 = 2 \times 10^{-26}$ kg m^{-3}.

   (a) Show that this Universe is closed and determine the value of k. [2 marks]

   (b) What will the scale factor and density be when the Universe reaches its maximum size? [3 marks]

   (c) How would this Universe evolve if $\Omega_\Lambda = 0.3$ rather than zero? (You can address this by tabulating a few values for H as a function of a.) [5 marks]

2. If decoupling occurred 3×10^5 years after the Big Bang, when the temperature of the Universe dropped to 3000 K, then, assuming a flat universe

   (a) give formulae for the evolution of the scale factor and the temperature with time, bearing in mind that the early Universe was radiation-dominated, [5 marks]
   (**Warning:** Consider carefully how your $a(t)$ expression should be normalised. Remember that the radiation-dominated era does not extend to the present day!)

   (b) calculate the radius of the particle horizon at the time of decoupling in comoving units (i.e. in Mpc *at the present epoch*) [5 marks]
   (**Hint:** To get full marks you will need to use the R-W metric and remember that a light ray has $ds = 0$.)

3. Adopting the current consensus cosmology ($\Omega_m = 0.3, \Omega_\Lambda = 0.7$, and $H_0 = 70$ km s^{-1} Mpc^{-1})
   (a) Calculate (i) the thermal pressure of baryonic matter, and (ii) the radiation pressure, at $z = 1000$, stating your assumptions clearly. [5 marks]

   (b) Explain the implications of the comparison between these two numbers for the behaviour of the Jeans Mass at decoupling. [5 marks]

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