THE COSMIC BARYON BUDGET

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ABSTRACT

We present an estimate of the global budget of baryons in all states, with conservative estimates of the uncertainties, based on all relevant information we have been able to marshal. Most of the baryons today are still in the form of ionized gas, which contributes a mean density uncertain by a factor of about 4. Stars and their remnants are a relatively minor component, comprising for our best-guess plasma density only about 17% of the baryons, while populations contributing most of the blue starlight comprise less than 5%. The formation of galaxies and of stars within them appears to be a globally inefficient process. The sum over our budget, expressed as a fraction of the critical Einstein–de Sitter density, is in the range $0.007 \lesssim \Omega_B \lesssim 0.041$, with a best guess of $\Omega_B \sim 0.021$ (at Hubble constant 70 km s⁻¹ Mpc⁻¹). The central value agrees with the prediction from the theory of light element production and with measures of the density of intergalactic plasma at redshift $z \sim 3$. This apparent concordance suggests that we may be close to a complete survey of the major states of the baryons.

Subject headings: cosmology: observations — elementary particles — galaxies: fundamental parameters

1. INTRODUCTION

The evolution of structure in the expanding universe has redistributed the baryons from a nearly smooth plasma at the time of light element nucleosynthesis to a variety of states—condensed, atomic, molecular, and plasma—in clouds of gas and dust, planets, stars, and stellar remnants, that are arranged in the galaxies, in groups and clusters of galaxies, and in between. The amounts of baryons in each state and form at low redshift can be compared to what is observed at higher redshift and to the total cosmic abundance predicted by the theory of element production in the early universe. Budget estimates must be informed by ideas of how structure evolves as well as by observations, and the budget in turn is a test of these ideas. Knowledge of the baryon budget is an essential boundary condition for analysis of how structure formed and the nature of cosmic dark matter.

Advances in observations allow reasonably sound estimates of the amounts of baryons present in a considerable variety of forms. In a previous discussion (Fukugita, Hogan, & Peebles 1996), we presented a picture of cosmic evolution suggested by the results of budget estimates. In this paper we give details of the budget calculations and update them using our choices for the current best knowledge relevant to the calculation. Persic & Salucci (1992), Gnedin & Ostriker (1992), and Bristow & Phillipps (1994) estimate baryon abundances with different emphases; the results are compared in § 5.1.

The main focus of this paper is the baryon budget at low redshift; our accounting is presented in § 2. The low-redshift budget can be compared to the situation at $z \sim 3$, where quasar absorption lines allow a comprehensive accounting of the diffuse components. We comment on this in § 3. The budgets are compared to the constraint from light element

production in § 4, and the implications for galaxy and structure formation are summarized in § 5.

We write Hubble's constant as

$$H_0 = 100 \ h \ \text{km s}^{-1} \ \text{Mpc}^{-1}$$
 . (1)

Where not explicitly written, we use solar units for mass and luminosity and megaparsecs for the length unit.

2. The baryon budget at $z \approx 0$

2.1. Stars and Remnants in Galaxies

Stars in high surface density galaxies are the most prominent location for baryons. For our purpose, it is reasonable to imagine that galaxies contain two distinct stellar populations, an old spheroidal component and a disk component consisting of generally younger stars, with a mix depending on galaxy type. Elliptical galaxies lack a significant disk component, and irregular (Im) galaxies are at the opposite extreme, having small or insignificant bulges. The formation of these two components seems to follow different histories, so it is appropriate to count baryons in stars divided into these two categories. The mass density for each component is obtained as

$$\rho(\mathrm{sph}, \, \mathrm{disk}) = \mathcal{L}_B f_B(\mathrm{sph}, \, \mathrm{disk}) \langle M/L_B \rangle_{\mathrm{sph}, \, \mathrm{disk}} , \qquad (2)$$

where \mathscr{L} is the mean luminosity density, f_B is the fraction of the luminosity density produced by the spheroid or disk component, and $\langle M/L_B \rangle$ is the mass-to-light ratio for each component including the stars and star remnants. The suffix B refers to luminosities measured in the B band, our choice for a standard wavelength band. Although the stellar population of irregular galaxies is similar to that of the disk component, we treat the former separately to emphasize its distinct role in the luminosity density at low redshift and a slightly smaller mass-to-light ratio because of its younger mean age.

TABLE 1
GALAXY PARAMETERS

Parameter	Е	S0	Sab	Sbc	Scd	Irr
Bulge fraction $\kappa(r)$	1	0.75	0.40	0.24	0.10	0
	1	0.64	0.33	0.16	0.061	0
	0.11	0.21	0.28	0.29	0.045	0.061
H I typical mass $(10^9 M_{\odot})$	0	0.64	2.5	4.0	2.9	1.8
H ₂ /H I		2.5	1.0	0.63	0.25	0.06

2.1.1. Luminosity Fractions

We consider first the luminosity fractions f in equation (2). In Table 1 we show the bulge fraction of galaxies in luminosity, $\kappa = L_{\rm bulge}/(L_{\rm bulge} + L_{\rm disk})$, for each morphological type, from the bulge-disk decomposition of Kent (1985), based on his accurate Gunn r-band CCD photometry. We take the median of the distribution presented by Kent. The bulge fraction in the r band is converted into that in the B band using the color transformation law B - r = 1.19 for elliptical galaxies and bulges (Fukugita, Shimasaku, & Ichikawa 1995). The values of κ thus obtained in the B band are consistent with the result of Whitmore & Kirshner (1981), although the latter sample is not as large as that of Kent, and the decomposition, based on photography, may be less accurate. The bulge fraction is well correlated with the morphology (Morgan 1962; Dressler 1980; Meisels & Ostriker 1984; Kent 1985), but the morphologies at given bulge fraction overlap neighboring classes.

Row 3 in Table 1 gives our adopted distribution of morphologies, expressed as the fraction μ_t of the mean luminosity density contributed by galaxies of morphological type t. Postman & Geller (1985) give E:S0:S + Irr = 0.12:0.23:0.65. This ratio is consistent with the Tinsley mix (Tinsley 1980), which subdivides the spiral fraction into further subclasses, while E and S0 are not separated. The luminosity fractions in equation (2) are obtained by combining the two results:

$$f_B(\text{sph}) = \sum \mu_t \kappa_t = 0.385 ,$$

 $f_B(\text{disk}) = \sum \mu_t (1 - \kappa_t) = 0.549 ,$
 $f_B(\text{irr}) = \mu_{\text{irr}} = 0.061 .$ (3)

The sum in the second expression is from S0s through spirals. Because the mass in S0 discs is so small, we make little error in treating the disks of spirals and S0s as the same population. The separation into each morphological type is ambiguous, and the classification depends much on the author. To model such error, we recalculate the fractions f after reclassifying $\frac{1}{3}$ of the galaxies in each type systematically to the neighboring later type, or else to the neighboring earlier type. This produces the range (0.324–

0.459):(0.599-0.499):(0.076-0.041). When we use a morphological fraction or a bulge-to-disk ratio taken from other authors, we find the luminosity fractions f fall within this range in most cases. In particular, when translated into the V passband, these estimates agree with the survey of Schechter & Dressler (1987). This spread of values of the μ_t is taken into account in our assignment of errors in estimates of the mean mass density.

2.1.2. The Mean Luminosity Density

Recent measurements of the mean luminosity density contributed by high surface brightness galaxies at moderately low redshift are summarized in Table 2. There is a substantial spread of estimates of the parameters ϕ^* and M^* in the Schechter function, but the total luminosity density, $\mathcal{L} = \phi^* L^* \Gamma(\alpha + 2)$, is better defined; most values are in the range $\mathcal{L}_B = (1.8-2.2) \times 10^8 \ h \ L_{\odot} \ \text{Mpc}^{-3}$. The exception is the significantly lower number from the APM-Stromlo survey (Loveday et al. 1992). We adopt

$$\mathcal{L}_B = (2.0 \pm 0.2) \times 10^8 \, h \, L_{\odot} \, \text{Mpc}^{-3} ,$$

$$(M/L_B)_{\text{crit}} = (1390 \pm 140)h . \tag{4}$$

The second expression is the corresponding value of the mass-to-light ratio in the Einstein—de Sitter model.

2.1.3. Mass-to-Light Ratio of the Spheroid Component

The mass-to-light ratio of stars and their remnants in spheroid populations can be inferred from star population synthesis calculations, assuming that the stars are close to coeval (so that the integrated B luminosity is not much affected by recent episodes of star formation) and the spheroid stars in our neighborhood are representative. Within these assumptions, the most serious uncertainty in the calculation is the lower cutoff of the initial mass function (the IMF). In the past, the cutoff was somewhat arbitrary, between 0.1 M_{\odot} and 0.2 M_{\odot} , and the choice seriously affected the estimate of M/L. Thus, for the Salpeter IMF Mdiffers by 24% between these two choices, while L is virtually unaffected. This uncertainty for the local IMF has been removed by the recent advance in observation of M subdwarf star counts with the Hubble Space Telescope; it has been shown that the IMF of the local Galactic stars has

TABLE 2
LUMINOSITY FUNCTION PARAMETERS

Author	M*(B)	α	ϕ^* (0.01 h^3 Mpc ⁻³)	$\mathcal{L}(B)$ (108 $h~L_{\odot}~Mpc^{-3}$)
Efstathiou et al. (1988)	-19.68	-1.1	1.56	1.93
Loveday et al. (1992)	-19.50	-0.97	1.40	1.35
da Costa et al. (1994)	-19.5	-1.20	1.5	1.71
Marzke et al. (1994)	-18.8	-1.0	4.0	2.1
Lilly et al. (1995)	-19.36	-1.03	2.48	2.18
Ellis et al. (1996)	-19.20	-1.09	2.6	2.05
Zucca et al. (1997)	-19.61	-1.22	2.0	2.2

a turnover at about 0.4 M_{\odot} (Gould, Bahcall, & Flynn 1996; hereafter GBF), and the integral over the GBF mass function converges without an arbitrary cutoff. For $M > M_{\odot}$, the GBF mass function agrees well with the Salpeter function. Using the GBF mass function for $M < M_{\odot}$, instead of the Salpeter mass function with x = 1.35 (where x is the index of the IMF $dn/dM \propto M^{-[1+x]}$), the mass integral is 0.70 times that obtained with the Salpeter mass function cutoff at 0.15 M_{\odot} . The light in the blue-visual bands contributed by stars less massive than 1 M_{\odot} is small; from Table 4 of Tinsley & Gunn (1976), we estimate that the V light integral decreases only 2% with the use of the GBF mass function. Therefore, the mass-to-light ratio with the GBF mass function is $(M/L_V)_{GBF} = 0.72(M/L_V)_{Salpeter}$, where the latter ratio is computed for the cutoff at $0.15\dot{M}_{\odot}$. Charlot, Worthey, & Bressan (1996) have given a compilation of mass-to-light ratios from various stellar population synthesis calculations. They show that M/L_V is reasonably consistent among authors when the lower mass cutoff is fixed. From the Charlot et al. (1996) compilation of population synthesis calculations, the GBF IMF, and the conversion formula discussed above, $(M/L_V)_{GBF} = (4.0 \pm 0.3) + 0.38(t_G - 10 \text{ Gyr}), \text{ where } t_G \text{ is}$ the age of the spheroid and the error represents the modeldependent uncertainty. Using the Charlot et al. (1996) synthetic calculation of the B-V color, we obtain $M/L_B=$ $(5.4 \pm 0.3) + 0.7(t_G - 10 \text{ Gyr})$. For $t_G = 12 \pm 2 \text{ Gyr}$, we have $M/L_V = 4.0 - 5.9$, or

$$M/L_R = 5.4 - 8.3$$
 (5)

The M/L ratio in equation (5) may be compared to the values inferred from the kinematics of the nuclear regions of elliptical galaxies. Van der Marel (1991) finds $M/L_R = (6.64 \pm 0.28) \ h$ (Johnson R), which translates to $M/L_V = (7.72 + 0.33) \ h$ or

$$M/L_R = (9.93 + 0.42)h$$
, (6)

using $\langle V-R_J\rangle=0.68$, corresponding to the average color $\langle B-V\rangle=0.92$ for the 37 elliptical galaxies he used $(\langle B-V\rangle_{\odot}=0.65$ and $\langle V-R_J\rangle_{\odot}=0.52$ are also used). There is excellent consistency between equations (5) and (6) if the Hubble constant is in the range h=0.6-0.8 and spheroid populations dominate the mass of these nuclear regions.

Dynamical measures show that M/L depends on the luminosity in elliptical galaxies (Kormendy 1986); van der Marel (1991) finds $M/L \propto (L/L^*)^{0.35 \pm 0.05}$. This manifestation of the color-magnitude relation of early-type galaxies probably reflects the effect of metallicity and perhaps also of dark matter halos. In the former case, we estimate that this dependence may reduce the effective mean value of M/L weighted by the contribution to the luminosity density by 17%, and we accordingly reduce the lower end of the allowed range by this amount.

We conclude that a good estimate of the mass-to-light ratio of stars and their remnants in spheroids is

$$M/L_B$$
(spheroids) = $6.5^{+1.8}_{-2.0}$. (7)

2.1.4. Mass-to-Light Ratio of the Disk Component

One way to estimate the mass-to-light ratio of the disk stars is to use estimates of the mass and luminosity column densities from surveys of stars and remnants in the solar neighborhood. Commonly used values are $\Sigma_M \simeq 45~M_{\odot}$

pc⁻² (stars and stellar remnants) and $\Sigma_{L_V} \simeq 15~L_{\odot}~{\rm pc}^{-2}$ (Binney & Tremaine 1987; Bahcall & Soneira 1980). GBF find that the mass column density must be reduced to $\Sigma_M \simeq 27.3~M_{\odot}~{\rm pc}^{-2}$ because of the turnover of the luminosity function toward the faint end, as noted above. With this new value, we have $M/L_V \simeq 1.82$ or $M/L_B \simeq 1.50$ for the disk with $B-V \simeq 0.44$ (de Vaucouleurs & Pence 1978). This is not very different from the early estimate of Mihalas & Binney (1981), $M/L_B \simeq 1.2$.

The solar-neighborhood value may be compared with M/L_B from population synthesis calculations. The Tinsley (1981) 10 Gyr model with a constant star formation rate gives $M/L_B=1.2$ after a 20% upward correction in the mass integral of the IMF to match GBF. Shimasaku & Fukugita's (1998) gas infall model (with effective disk age ≈ 10 Gyr) gives $M/L_B=1.9$ after a 40% reduction in M to correct the IMF. From a study of rotation curves for spiral galaxies, Persic & Salucci (1992) find $M/L_b=1.24$ h. We conclude that there is reasonably small scatter among these different approaches to the disk mass-to-light ratio, and we adopt

$$(M/L_R)_{\rm disk} = 1.5 \pm 0.4$$
 . (8)

There is good evidence that stars in gas-rich irregular (Im) galaxies are young. (For reviews, see Fukugita et al. 1996; Ellis 1997). If the mean age of irregular galaxy stars is ≈ 5 Gyr, the *B* luminosity is 0.5 mag brighter and the mass in star remnants is about 10% smaller than for a 10 Gyr old disk. These corrections reduce M/L in equation (8) to $M/L_B = 1.1$ for irregular galaxies. A population synthesis model for an age of 8 Gyr (Shimasaku & Fukugita 1998) also gives $M/L_B = 1.1$ after the correction to the IMF. We take

$$(M/L_B)_{\rm irr} = 1.1 \pm 0.25$$
 (9)

2.1.5. Baryons in Stars and Remnants

With the above numbers, the mean mass densities in our three classes of stars and their associated remnants are

$$\Omega_{\text{spheroid stars}} = (0.00180^{+0.00121}_{-0.00085})h^{-1},$$
(10)

$$\Omega_{\text{disk stars}} = (0.00060^{+0.00030}_{-0.00024})h^{-1},$$
(11)

$$\Omega_{\text{stars in Irr}} = (0.000048^{+0.0000033}_{-0.000026})h^{-1},$$
 (12)

in units of the critical density $\rho_{\rm crit}=2.775\times 10^{11}~h^2~M_{\odot}$ Mpc⁻³. Here and below the upper (or lower) errors are obtained by taking all errors in each step to maximize (or minimize) the result. In this sense, the error ranges are conservative. The numbers in equations (10)–(12) depend through the M/L calculation on age estimates, and hence depend on the Hubble constant h and other cosmological parameters in a complicated way. If dynamics were used to estimate M/L, the estimates of Ω would not depend on h, but in that case $h\approx 0.7$ would be needed for dynamics to agree with the synthesis calculations. Either way, consistency holds for $h\approx 0.7$ in a low-density universe.

2.2. Atomic and Molecular Gas

From H I 21 cm surveys, Rao & Briggs (1993) find $\Omega_{\rm H\,I}=(1.88\pm0.45)\times10^{-4}~h^{-1}$ (see also Fall & Pei 1993; Zwaan et al. 1997). A 32% addition (for 24% helium mass fraction) to the Rao-Briggs value for helium yields

$$\Omega_{\text{atomic}} = (0.00025 \pm 0.00006)h^{-1}.$$
(13)

This result is quite secure, because absorption line studies show directly that most of the H I is in systems with column densities high enough to detect in 21 cm emission. These columns are also generally high enough to resist photoionization, so we are counting here mostly regions dominated by neutral atomic gas.

The ratios H_2/H I of mass in molecular and atomic hydrogen listed in Table 1 for each morphological type are median values taken from the compilation of Young & Scoville (1991), who base the molecular hydrogen masses on CO observations. The representative masses in atomic hydrogen in Table 1 are from Roberts & Haynes (1994). Combining these numbers with the distribution μ_t in morphology, weighted by H I mass, we obtain the global ratio in galaxies $\rho_{H_2}/\rho_{H_1} = 0.81$, and from equation (13)

$$\Omega_{\rm H_2} = (0.00020 \pm 0.00006)h^{-1} .$$
(14)

2.3. Baryons in Clusters of Galaxies

The cluster mass function is approximated as (Bahcall & Cen 1993)

$$n_{\rm cl}(>M) = 4 \times 10^{-5} \ h^3 \left(\frac{M}{M^*}\right)^{-1} \exp\left(-\frac{M}{M^*}\right) \rm Mpc^{-3}$$
, (15)

where $M^* = (1.8 \pm 0.3) \times 10^{14} \, h^{-1} \, M_{\odot}$, and M is the total gravitational mass within a sphere of radius 1.5 h^{-1} Mpc (the Abell radius) centered on the cluster. At the Abell radius, the mass distribution is close to dynamical equilibrium. The envelope of matter around the cluster at greater distances merges into the large-scale structure, and we take this matter to be included in the field (as discussed in § 2.4). We define clusters as objects with mass $M > 10^{14} \, h \, M_{\odot}$. The integral $\int dMM \, dn_{\rm cl}/dM$ gives the mean mass density in clusters,

$$\rho_{\rm cl} = (7.7^{+2.5}_{-2.2}) \times 10^9 \ h^2 \ M_{\odot} \ {\rm Mpc^{-3}} \ .$$
 (16)

The contribution of this gravitational mass to the density parameter is

$$\Omega_{c1} = 0.028^{+0.009}_{-0.008}$$
 (17)

Intracluster plasma masses are well determined from X-ray observations (Fabricant et al. 1986; Hughes 1989; White et al. 1993). The ratio of X-ray-emitting gas mass to gravitational mass within the Abell radius from the survey by White & Fabian (1995) is

$$(M_{\rm H\,II}/M_{\rm grav})_{\rm cl} = (0.056 \pm 0.014)h^{-3/2}$$
 (18)

In the White & Fabian (1995) sample, this ratio shows no correlation with cluster mass. The value given in equation (18) is independently verified by Myers et al. (1997), $(M_{\rm H\,II}/M_{\rm grav})_{\rm cl}=(0.061\pm0.011)h^{-1}$ from the measurement of the Sunyaev-Zeldovich effect in three clusters. The product of equations (17) and (18) is

$$(\Omega_{\rm H\,II})_{\rm cl} = (0.00155^{+0.00100}_{-0.00072})h^{-1.5} \,. \tag{19}$$

This is the contribution to the baryon budget by intracluster plasma.

Let us estimate the baryons in stars in galaxies in clusters. Although these stars are included in the estimate for stars in § 2.1, it is useful to compare their mass with the plasma mass, which we can derive from the cluster mass-to-light ratio. In the discussion of the Coma cluster by White et al. (1993), straightforward applications of galaxy velocity dis-

persions or the X-ray pressure gradient give $(M_{\rm grav}/L_B)_{\rm cl} \sim$ 370 h, or values as large as 500 h if the analysis is constrained by models from numerical simulations of cluster formation. The CNOC value (Carlberg et al. 1996) transformed to the B band by $(B-r)_{\odot}=0.65$ and $\langle B-r\rangle=1.03\pm0.1$ for the average color of S0 galaxies, after a passive evolutionary correction for early-type galaxies, is somewhat larger, 560 h. However, the CNOC luminosity density is correspondingly smaller than our adopted value (eq. [4]), and it is the product of this with M/L that matters in deriving most global quantities; moreover, we suspect that the product is more reliably estimated, since in the end we are estimating the luminosity density in cluster galaxy stars, which does not depend on a mass estimate. We therefore adopt a central value,

$$(M_{\rm grav}/L_B)_{\rm cl} = (450 \pm 100)h$$
 . (20)

For cluster galaxies (Dressler 1980), we adopt the composition within the Abell radius

$$\mu(E) = 0.21$$
, $\mu(S0) = 0.44$, $\mu(S) = 0.35$. (21)

(This is somewhat richer in early-type galaxies than the estimate given by Schechter & Dressler 1987, but their sample extends beyond the Abell radius.) The mass-to-light ratios in equations (7) and (8) and the bulge-to-disk ratios in Table 1 give $(M/L)_{\rm star} = 4.5 \pm 1$. The ratio to equation (20) gives $h \ M_{\rm stars}/M_{\rm grav} = 0.010^{+0.005}_{-0.004}$. The product with equation (17) is the contribution to the density parameter by stars in clusters,

$$(\Omega_{\text{stars}})_{\text{cl}} = (0.0003^{+0.0003}_{-0.0002})h^{-1},$$
 (22)

about 10% of the total in equations (10)–(12).

2.4. Baryons in Groups

Plasma associated with galaxies outside the great clusters makes a significant and still quite uncertain contribution to the baryon budget. The detected X-ray emission from plasma in groups is softer than for clusters, and softest for groups dominated by late-type galaxies. It is a reasonable presumption that the absence of detections often represents a lower virial temperature, rather than the absence of plasma.

We follow two approaches in estimating the mass in plasma associated with galaxies outside clusters. In the first, we start with a characteristic value for the ratio of plasma mass to gravitational mass in groups with X-ray detections. We apply this ratio to all galaxies, on the assumption that all field galaxies are likely to have inherited a similar plasma mass, to estimate the density in the warm phase (§ 2.4.1). To this we add an estimate from quasar absorption of the plasma mass in low surface density cool clouds that certainly are not counted in the X-ray measurements (§ 2.4.2). The second approach (§ 2.4.3) assumes that the plasma-to-gravitational mass ratio is the same in clusters and the field.

2.4.1. Warm Plasma in Groups

Some surveys are available from low-energy X-ray observations with ROSAT (Mulchaey et al. 1996). The X-ray emission from 18 groups with total mass ranging from 1.2 to $8.3 \times 10^{13}~h~M_{\odot}$ corresponds to plasma mass fractions ranging from 0.004 $h^{-3/2}$ (H97) to 0.09 $h^{-3/2}$ (NGC 4261). The average is

$$(M_{\rm H\,II}/M_{\rm gray})_{\rm group} = (0.022 \pm 0.005)h^{-3/2}$$
, (23)

significantly below the number for clusters (eq. [18]). In fact, the baryon fraction shows a trend of increasing with the group mass, approaching the cluster value at the highmass end. This could be because groups are intrinsically poorer in plasma, or because much of the plasma is cooler and so escapes detection as an X-ray source. The latter is in line with the shallower gravitational potential wells in groups. The cool plasma clouds detected by Lyα resonance absorption (§ 2.4.2) similarly are not detected as X-ray sources. Thus, the plasma identifiable from its discrete X-ray emission might be considered a lower limit to the net plasma associated with groups of galaxies.

To convert equation (23) into a mean baryon density, we need the mean gravitational mass associated with field galaxies (which are almost always in groups). The following measures may be compared. First, we can extrapolate the Bahcall & Cen (1993) mass function (eq. [15]), which they determined for $M>10^{13}~h~M_{\odot}$, to systems with the mass characteristic of galaxies, $M\sim 10^{12}~h~M_{\odot}$. The result of integrating this mass function from $M=10^{12}~h~M_{\odot}$ to $M=10^{14}~h~M_{\odot}$ is

$$\Omega = 0.12 \pm 0.02 \ . \tag{24}$$

The cutoff is the characteristic mass of an L^* galaxy, the minimum for a group.

Second, we have dynamical mass measures from analyses of systems of galaxies on scales smaller than about $10\ h^{-1}$ Mpc and outside the rich clusters. The survey of results of these analyses by Bahcall, Lubin, & Dorman (1995) indicates that $M_{\rm grav}/L \simeq (200^{+100}_{-50})h$. With the mean luminosity density in equation (4), this gives

$$\Omega = 0.14^{+0.10}_{-0.05} \,. \tag{25}$$

Third, we can use the luminosity density, scaling from M/L calibrated in the great clusters, by taking into account the difference in luminosities from the difference in morphological mixes (see also Carlberg et al. 1997). The small scatter in the color-magnitude relation for ellipticals and the spheroid components of spirals suggests that these stars formed early, so it is reasonable to assume that the ratio of spheroid luminosity to gravitational mass is the same in clusters and the field. It would follow that the cosmic mean mass-to-spheroid light ratio is $(M/L_B)_{cos} = (M_{grav}/L_B)_{cl}$ $f(\mathrm{sph, cos})/f(\mathrm{sph, cl}) = (270 \pm 60)h$, where $f(\mathrm{sph, cos})$ is the cosmic mean spheroid population luminosity fraction (eq. [3]), the value in clusters, f(sph, cl) = 0.64, follows from equation (21), and $(M_{\rm grav}/L_B)_{\rm cl}$ is from equation (20). With equation (4), we have $\Omega=0.19^{+0.07}_{-0.05}$. This, however, gives the cosmic mean including clusters; for comparison to equation (25), we ought to convert to the value outside rich clusters. The ratio of equation (16) to equation (20) is the mean luminosity density provided by the clusters, $\mathcal{L}_{\text{cl}} =$ $0.09\mathcal{L}$. The spheroid luminosity fraction in the field is

$$f_{\text{field}}(\text{sph}) = (\mathcal{L}f_{\text{cos}} - \mathcal{L}_{\text{cl}}f_{\text{cl}})/(\mathcal{L} - \mathcal{L}_{\text{cl}}) = 0.94f_{\text{cos}}$$
. (26)

The assumption that the mass-to-spheroid light ratio is universal thus indicates that the density parameter in gravitational mass outside the Abell radii of the great clusters is

$$\Omega = 0.18^{+0.07}_{-0.05} \,. \tag{27}$$

Despite the substantial uncertainties in each of these arguments, we are encouraged by the consistency of equations (24), (25), and (27) to conclude that the density parameter in gravitational mass that clusters with galaxies on

scales of $\leq 10 h^{-1}$ Mpc is likely to be in the range

$$\Omega = 0.15^{+0.10}_{-0.05}, \tag{28}$$

and that the spheroid-to-dark matter and baryon-to-dark matter ratios indeed do not vary widely between clusters and the field. As the evidence has indicated for some time (Peebles 1986), and as has been widely noted in recent years, this low density parameter universe offers a natural interpretation for a variety of observations. Recent examples include the age/distance scale relation, the abundance of cluster baryons (White et al. 1993), the growth rate of correlation functions (Peacock 1997), the growth rate of the cluster mass function (Bahcall, Fan, & Cen 1997), and preliminary indications from the distant supernova Hubble diagram (Garnavich et al. 1998; Perlmutter et al. 1998).

The product of equations (23) and (28) is the estimate of total plasma identified by X-ray emission in groups,

$$(\Omega_{\rm H\,II})_{\rm group} = (0.003^{+0.004}_{-0.002})h^{-3/2}$$
 (29)

We remark that this estimate decreases by only 20% when we explicitly incorporate the trend that the baryon fraction increases as the group mass, $(M_{\rm H\,II}/M_{\rm grav})_{\rm group} \sim 0.056~h^{-3/2}(M_{\rm group}/0.6\times10^{14}~M_{\odot})$ for $M_{\rm group}<0.6\times10^{14}~M_{\odot}$. As we have noted, an alternative interpretation is that the trend is only apparent, a result of less efficient detection of plasma around cooler groups.

2.4.2. Plasma in Cool Low Surface Density Clouds

In addition to the hot plasma detected in X-rays, there is a considerable amount of plasma in cool, thermally stable, photoionized clouds with temperatures less than about 20,000 K. These cool low column density clouds are detected by the Lva resonance absorption line in quasar spectra. They may be found as far as 5 Mpc from a galaxy, and generally seem to avoid both the voids and strong concentrations of galaxies (Shull, Stocke, & Penton 1996). This is consistent with the view that such clouds were left behind in the gravitational assembly of groups of galaxies; those that would have belonged to cluster members were incorporated and shock heated by the assembly of the clusters, joining the hot phase. In our budget for plasma in groups, we thus use the sum of the mean baryon densities in these clouds and in cluster baryons detected by X-ray emission (eq. [29]).

The estimate of the baryon abundance in such clouds is very sensitive to model details such as ionization and geometry. Let us assume as an illustrative model that the clouds are uniform-density, isothermal spheres in photoionization equilibrium, with size on the order of 100 kpc, determined empirically by common absorption between two nearby lines of sight. Then, the mass of the clouds that yield the neutral hydrogen column density $N_{\rm H\,I}$ are estimated, following Shull et al. (1996), to be

$$M_{\rm Ly\alpha} = 7.1 \times 10^8 \ M_{\odot} \ J_{-23}^{1/2} \left(\frac{T}{2 \times 10^4}\right)^{0.375} \times N_{14}^{1/2} \left(\frac{R}{100 \ \rm kpc}\right)^{5/2} \left(\frac{3 + \alpha}{4.5}\right)^{-1/2}, \tag{30}$$

where J_{-23} is the ionizing UV flux at the Lyman limit in units of 10^{-23} ergs s⁻¹ cm⁻² sr⁻¹ Hz⁻¹, with a power index α , T is the gas temperature, N_{14} is the neutral hydrogen column density in units of 10^{14} cm⁻², and R is a typical cloud size. The neutral hydrogen fraction

 $x = n_{\rm H\,I}/n_{\rm H} \simeq 6 \times 10^{-5} J_{-23}^{-1/2} [T/(2 \times 10^4)]^{-0.375} N_{14}^{1/2} (R/100~{\rm kpc})^{-1/2} [(3+\alpha)/4.5]^{1/2}.$ With the detection frequency of the absorbers at $z \simeq 0$, $dN_c/dz = \phi_0(\pi R^2)c^2/H_0 \approx 86$, the mass density of the clouds is

$$\Omega_{\text{Ly}\alpha} = \frac{\phi_0 M_c}{\rho_{\text{crit}}} \simeq 0.00234
\times \frac{J_{-23}^{1/2} [T/(2 \times 10^4)]^{0.375} N_{14}^{1/2} (R/100 \text{ kpc})^{1/2}}{h[(3 + \alpha)/4.5]^{1/2}} .$$
(31)

This assumes spherical clouds. If the clouds are oblate, the density decreases by the aspect ratio, which we assume to be between $\frac{1}{5}$ and 1.

Direct simulations of cloud formation and absorption spectra permit a much more detailed, though model-dependent, picture of the clouds. It has been demonstrated that these simulations give good fits to the observations at $z \approx 3$ (see below), but they do not yet yield useful approximations to the situation at z = 0. Thus, the direct estimates of the density parameter in this component remain quite uncertain.

2.4.3. Scaling from Clusters to Groups

As an alternative to adding the previous two directly observed components (warm X-ray emitting gas and cool Ly α absorbing gas), we can estimate the total plasma in groups from the assumption that it has the same ratio to dark matter as in the rich clusters. That is, the scaling used to arrive at the density parameter in gravitational mass in the field from spheroid luminosity (eq. [27]) readily generalizes to a scaling estimate of the density parameter in plasma associated with groups. The product of equation (18) for the ratio of plasma to gravitational mass in clusters and equation (28) for the density parameter in the field yields

$$(\Omega_{\rm H\,II})_{\rm field} = (0.008^{+0.010}_{-0.004})h^{-3/2}$$
 (32)

In a calculation along similar lines, Carlberg et al. (1997) find a similar result, $(\Omega_{\rm H\,II})_{\rm field} = (0.01-0.014)h^{-3/2}$. The

central value in equation (32) agrees with the upper bound from the sum of the more direct estimates in equations (29) and (31).

Equation (32) assumes that a fair sample of plasma relative to gravitational mass is assembled and preserved in the great clusters. If disks were less common in clusters because star formation has been suppressed, equation (32) would include baryons in disks in the field, but the correction is small because the baryon mass in disk stars is small. It is not thought that cluster cooling flows or winds have seriously depleted the cluster plasma mass. In a more direct argument, Renzini (1997) finds that the iron and gas content of clusters and groups indicate that the clusters indeed have close to the global star formation efficiency, and that the corresponding enriched gas from groups has been ejected into intergalactic space, presumably ending up in the form discussed in §§ 2.4.1 and 2.4.2.

2.5. Summary of the Low-Redshift Budget

Our estimates of the baryon budget at low redshift, expressed as fractions of the critical Einstein–de Sitter value, are listed in lines 1 to 7 in Table 3. We rewrite the Hubble constant from equation (1) as $H_0 = 70 \ h_{70} \ \text{km s}^{-1}$ Mpc⁻¹ to center it on a more likely value. The central values in the table are computed from the central values of the parameters entering the estimate. An upper bound is obtained by systematically choosing each parameter at the end of its reasonable-looking range that maximizes the contribution to the mean mass density, and likewise for the lower bound.

Line 7a of Table 3 is an estimate of the X-ray-detected plasma around groups (eq. [29]), line 7b is an estimate of the cooler component detected by resonance Ly α absorption in the trace H I fraction (eq. [31]), and the sum is an estimate of the plasma associated with groups of galaxies. Both entries are quite uncertain, and may well have missed a substantial amount of cool plasma. Consistent with this, the entry in line 7' from the scaling from the plasma mass in clusters of galaxies (eq. [32]) is larger. We suspect that line

TABLE 3
THE BARYON BUDGET

Component	Central	Maximum	Minimum	Gradea					
Observed at $z \approx 0$									
 Stars in spheroids Stars in disks Stars in irregulars Neutral atomic gas Molecular gas Plasma in clusters Warm plasma in groups Cool plasma Plasma in groups Sum (at h = 70 and z ≈ 0) 	$\begin{array}{c} 0.0026\ h_{70}^{-1} \\ 0.00086\ h_{70}^{-1} \\ 0.000069\ h_{70}^{-1} \\ 0.00033\ h_{70}^{-1} \\ 0.00030\ h_{70}^{-1} \\ 0.0026\ h_{70}^{-1.5} \\ 0.0056\ h_{70}^{-1.5} \\ 0.002\ h_{70}^{-1} \\ 0.014\ h_{70}^{-1} \\ 0.021 \end{array}$	$\begin{array}{c} 0.0043 \ h_{70}^{-1} \\ 0.00129 \ h_{70}^{-1} \\ 0.000119 \ h_{70}^{-1} \\ 0.000116 \ h_{70}^{-1} \\ 0.00037 \ h_{70}^{-1} \\ 0.00044 \ h_{70}^{-1.5} \\ 0.0115 \ h_{70}^{-1.5} \\ 0.003 \ h_{70}^{-1} \\ 0.030 \ h_{70}^{-1} \\ 0.041 \end{array}$	$\begin{array}{c} 0.0014\ h_{70}^{-1} \\ 0.00051\ h_{70}^{-1} \\ 0.000031\ h_{70}^{-1} \\ 0.000025\ h_{70}^{-1} \\ 0.00023\ h_{70}^{-1} \\ 0.0014\ h_{70}^{-1.5} \\ 0.0029\ h_{70}^{-1.5} \\ 0.00072\ h_{70}^{-1} \\ 0.0072\ h_{70}^{-1} \end{array}$	A A - A B C B					
Gas components at $z \approx 3$									
9. Damped absorbers	$0.0015 \ h_{70}^{-1} \\ 0.04 \ h_{70}^{-1.5} \\ \cdots$	$\begin{array}{c} 0.0027 \; h_{70}^{-1} \\ 0.05 \; h_{70}^{-1.5} \\ 0.01 \; h_{70}^{-1.5} \end{array}$	$\begin{array}{c} 0.0007 \; h_{70}^{-1} \\ 0.01 \; h_{70}^{-1.5} \\ 0.0001 \; h_{70}^{-1} \end{array}$	A – B B					
Abundances of:									
12. Deuterium 13. Helium 14. Nucleosynthesis	$0.04 \ h_{70}^{-2} \ 0.010 \ h_{70}^{-2} \ 0.020 \ h_{70}^{-2}$	$\begin{array}{c} 0.054 \ h_{70}^{-2} \\ 0.027 \ h_{70}^{-2} \\ 0.027 \ h_{70}^{-2} \end{array}$	$0.013 \ h_{70}^{-2} \dots \\ 0.013 \ h_{70}^{-2}$	A A 					

^a Confidence of evaluation, from A (robust) to C (highly uncertain).

7' is more reliable than the sum of lines 7a and 7b. In the sums over the budget, we use line 7' for the maximum and central values, and lines 7a and 7b instead of 7' for the minimum.

The true values for each line are not likely to be close to the maximum in all entries, or close to the minima in all entries, but we retain the whole range in quoting the sums in each column in line 8 because the sums are dominated by a few entries.

At $h_{70}=1$, our estimate of the baryons in stars and remnants is $\Omega=0.0035^{+0.0025}-0.0014$ times the critical density. This star mass is only about 17% of our sum over all detected baryons. In our budget, in the $2.2~\mu m~K$ band, 26% of the light is from disk and irregular galaxies, in agreement with the fraction of the star mass. (We use $\langle B-K\rangle=4.27$ for ellipticals and bulges, and 2.78 for disks and irregulars). In the B band, 61% of the mean luminosity density is from disks and irregular galaxies. This component represents only about 25% of the star mass and only about 4% of our estimated sum of all observed baryons.

2.6. Uncounted Components

We discuss here several mass components that are not counted in our budget because the observational evidence about them is still sketchy.

2.6.1. Warm Plasma in the Voids

There could be a very large baryon mass in plasma in the voids. Its emission may even have been observed; plasma at a temperature of $\approx 2 \times 10^6$ K fits a component of the diffuse soft X-ray background (Wang & McCray 1993). This might be produced locally by a relatively small amount of gas in the Galaxy (for a summary of evidence for and limits on hot gas around the Milky Way, see Moore & Davis 1994), or it could be produced by diffuse extragalactic gas with a mean density

$$(\Omega_{\rm H\,II})_{\rm IGM} = 0.2\zeta^{-1/2}C^{-1/2}h_{70}^{-3/2},$$
 (33)

where the emissivity parameter ranges from $\zeta = 1$ for solar to $\zeta = 0.1$ for primordial abundances, and the clumping parameter $C \equiv \langle n_e^2 \rangle / \langle n_e \rangle^2$ may be significantly greater than unity. Gas at this temperature at low redshift has few other detectable effects. For example, the COBE limit on the Compton distortion of the microwave background spectrum, $y < 1.5 \times 10^{-5}$ (Fixsen et al. 1996), is produced by a Hubble length of gas at a mean density $\Omega = (T/10^6)^{-1}$ h^{-1} , so the bound on the y-distortion is not useful here. On theoretical grounds, however, it is difficult to see how there could be much mass in void plasma. If primeval, it would be surprising if the voids contain little else in the form of baryons, especially as detected in diffuse helium absorption at $z \approx 3$ (see below). If blown in, it would be surprising if the plasma far exceeds the density of stars, and the required temperature in this case (to achieve sufficient gas velocity to refill the voids in a Hubble time) exceeds 10⁷ K, which does produce excessive anisotropy and spectral distortion in the background radiation.

2.6.2. *MACHOs*

The massive compact halo objects (MACHOs) detected as gravitational microlensing events could be stellar remnants, that is, baryons. The nature of the MACHOs is not known, and there is no secure estimate of their global abundance. However, they do seem to comprise a new population not otherwise accounted for.

Current results from the MACHO collaboration (Alcock et al. 1997) indicate that objects with masses comparable to that of the Sun (in one model, $0.5^{+0.3}_{-0.2} M_{\odot}$) may account for 20% to 100% of the dark mass in a standard spherical halo between the Milky Way and the Large Magellanic Cloud. This result must be taken with caution, however, because the experiment measures the MACHO mass column in just one direction in one halo, and the extrapolation to a global density is subject to many uncalibrated assumptions. The Galactic MACHO population may have asymmetries or the MACHOs may be concentrated relative to the global dark matter in the halo. For example, the actual mass of MACHOs inferred within 50 kpc (based on a spherical model, but without extrapolating to larger radii) is estimated at 68% confidence to be 13 to 32 \times 10¹⁰ M_{\odot} . This can be compared to the mass of the disk (about 6×10^{10} M_{\odot}). A reasonable estimate of the minimum global density of MACHOs, viewing them as a new Galactic stellar population, is to assume that all disk galaxies have the same ratio of MACHO to disk mass, taking the low end of the estimated range; this yields

$$\Omega_{\text{MACHO,min}} = 2.2\Omega_{\text{disks,min}} = 0.0011 \, h_{70}^{-1} ,$$
(34)

a minor entry in the budget. On the other hand, the data are consistent with all of the Galactic dark matter being in MACHOs, so a reasonable upper limit derives from assuming that MACHOs comprise 100% of the density parameter in gravitation mass in equation (28), giving $\Omega_{\text{MACHO},\text{max}} = 0.25$, and making MACHOs the dominant entry in the baryon budget. (At this level, one expects detectable effects due to microlensing of quasar continuum emission regions, e.g., Dalcanton et al. 1994).

Assessments of this serious uncertainty will be guided by advances in observational constraints on the nature and amount of the MACHOs, and perhaps also by advances in understanding processes of star formation and death that could produce a substantial mass in baryonic MACHOs without leaving an unacceptably large amount of debris.

2.6.3. Dwarf Galaxies and Low Surface Brightness Galaxies

The fainter end of the galaxy luminosity function has been a matter of debate for some time, but recent work with larger volume surveys (Lin et al. 1996; Zucca et al. 1997) indicates that the luminosity function is close to flat for at least 5 mag down from L* for the local field galaxies. There is some evidence for a rise of the fainter tail right after the shoulder of L* (Ellis et al. 1996), but this is visible only in a high-z sample selected in the blue band, for which small galaxies show extraordinary star formation activity without contributing much to the baryon budget. This means that the contribution of dwarf galaxies to the baryon budget is small, and the estimate from the local luminosity density is likely more secure. A rise in the luminosity function somewhat below L^* for cluster members has also been reported. (For the most recent literature, see Phillipps et al. 1997). The contribution to the mass density from these cluster dwarf galaxies cannot be dominant unless this sharp rise continues to very low mass, which is not likely, since it would violate measurements of cluster surface brightnesses.

The review by Bothun, Impey, & McGaugh (1997) indicates the importance of galaxies with surface brightnesses lower than the more readily detectable "normal" galaxies. It is not clear, however, that low surface brightness (LSB) galaxies make a significant contribution to the baryon

budget. LSB galaxies contribute to the budget by their stars and star remnants, neutral gas, and plasma. The 21 cm observations by Briggs (1997) indicate that there is not a significant contribution to the mean density of atomic hydrogen from gas-rich LSB galaxies in the field at distances of $\leq 10 \ h^{-1}$ Mpc. A direct constraint on diffuse plasma around LSB galaxies is much more difficult, but since LSB galaxies avoid the voids defined by the more visible galaxies, this is a problem of counting the plasma concentrated around galaxies outside the great clusters, as summarized in lines 7a, 7b, and 7' of Table 3. As we have noted, diffuse baryons initially associated with LSB galaxies that end up as cluster members would be counted in the X-ray measurements of $(\Omega_{\rm H\ II})_{\rm cl}$ (eq. [19]) and in the scaled value for the plasma in the field (eq. [32] and line 7' of Table 3). The mean luminosity density in the LSB sample considered by Sprayberry et al. (1997) is 15% of our adopted value (eq. [4]). The mean luminosity density from all stars not shrouded by dust, including those in systems with luminosities or surface brightnesses below detection thresholds, is constrained by measurements of the extragalactic contribution to the sky surface brightness. Absolute measurements in progress (Bernstein 1997) are capable of reaching $\sim 3\mathcal{L}$ (eq. [4]), but differential measurements of background fluctuations already constrain plausible new populations to contributing much less than \mathcal{L} (Dalcanton et al. 1997; Vogeley 1998). We cannot exclude the possibility that there is a significant mass in brown dwarfs or baryonic MACHOs in LSB galaxies, but we can note that since many of these galaxies seem to be in early states of evolution, they would not seem to have had much opportunity to have sequestered mass in dark stars, and that the integrated baryon content in present-day LSB galaxies likely is counted in the measures of diffuse gas at redshift $z \sim 3$, as discussed next.

3. THE BARYON BUDGET AT $z \approx 3$

There has been significant progress in understanding stellar populations at high redshift (e.g., Madau et al. 1996), but many issues remain open. It is fortunate for our purpose that stars are subdominant in the budget at the present epoch and likely are even less important at high redshift, so we can concentrate on the constraints on diffuse gas from quasar absorption line spectra.

The neutral hydrogen at $z \sim 3$ is predominantly in the high column density damped-Ly α absorbers (DLAs; Lanzetta, Wolfe, & Turnshek 1995 and references therein). The amount of neutral hydrogen in the DLAs increases with increasing redshift back to $z \sim 2$. Our adopted value for the density parameter in this form at 2 < z < 3 is from Storrie-Lombardi, Irwin, & McMahon (1996):

$$\Omega_{
m atomic}(z=3)$$

=
$$(0.0013 \pm 0.0003, 0.0020 \pm 0.0007) \left(\frac{h}{70}\right)^{-1}$$
. (35)

The two estimates are for $\Omega=0$ and $\Omega=1$, respectively, and include the mass of the accompanying helium. Because these systems are optically thick to ionizing radiation, there is no correction for an ionized fraction in the neutral gas. (The significant amount of mass in plasma in H II regions around young stars or in regions exposed to the intergalactic ionizing radiation are included in the forest component below). Equation (35) could be an underestimate if

extinction by dust in the gas suppressed the selection of quasars behind high column density absorbers (Fall & Pei 1993) or an overestimate if gravitational lensing enhanced the selection of lines of sight through dust-free, gas-rich absorbers.

Yet another uncertainty is the residency time of gas in DLAs. The large velocities in the DLAs (Prochaska & Wolfe 1997) indicate that the cloud masses could be depleted by winds only if there were considerable energy input from supernovae. The more likely scenario is that the density parameter in DLAs is decreasing at $z \le 2$ because the H I is being converted into stars. The H I mass in DLAs at z=3 is about half that in present-day stars (lines 1, 2, 3, and 9 in Table 3). This could mean that there is a significant mass in stars in DLAs and other young galaxies at z=3 and/or that intergalactic matter still is settling onto protogalaxies at z=3. However, these issues do not affect the budget regarded as a snapshot of conditions at z=3.

The dominant baryonic mass component is the Lya forest gas, detected by the trace neutral hydrogen in plasma that fills space as a froth at z = 3. The density can be estimated from CDM model simulations that give good fits to the observations taking into account the distributions of cloud shapes, sizes, flow velocities, and temperatures, although these estimates are still sensitive to the uncertain flux of ionizing radiation. Rauch et al. (1997) conclude that the baryon density parameter needed to correctly reproduce the statistical absorption (mostly due to clouds at H I surface density $\simeq 10^{13\pm1}~{\rm cm}^{-2}$) is $\Omega_{\rm H\,II}~h^2 > 0.017-0.021$, depending on cosmological model. Weinberg et al. (1997) quote a lower limit of $0.0125 \ h^{-2}$. Zhang et al. (1997), including a self-consistent analysis of the ionizing spectrum, arrive at a range for h = 0.5 of $0.03 < \Omega_b < 0.08$, with about half of this in the forest clouds. Smaller densities may be possible, however, because the same absorption can be produced by a higher H I fraction, caused by higher density contrast and lower gas entropy (Wadsley & Bond 1996; Bond & Wadsley 1997). Because of the unresolved issues, we assign the current estimates a "B" grade. In Table 3 we adopt the range given by Zhang et al. (1997), scaling by $h^{-3/2}$. For the lower limit, we include not the total density but just that in the forest clouds; this is appropriate for consistency, since the simulations predict a larger fraction of cooled baryonic matter than we infer from observations.

Cool plasma between the forest clouds has lower density and hence a lower neutral fraction and very low H I Lya resonance optical depth. The amount of plasma in this form is best probed by measurements of resonance absorption by the most abundant absorbing ion, singly ionized helium (Jakobsen et al. 1994; Davidsen, Kriss, & Zheng 1996.) High-resolution HST/GHRS quasar absorption line spectra of two quasars now permit the separation of the more diffuse component of the He II resonance absorption from the component in the H I Lyα forest clouds (Hogan, Anderson, & Rugers 1997; Reimers et al. 1997). The upper bound on intercloud gas density is derived from the maximum permitted mean He II Lyα optical depth allowed after subtracting the minimal contribution from the detected H I Lya forest clouds, while adopting the hardest ionizing spectrum allowed by the data, with opposite assumptions leading to the lower bound. (Note that the upper bound is on photoionized cool gas; hot gas with thermally ionized helium is not constrained by absorption, but is even less plausible at $z \approx 3$ than at $z \approx 0$.) These

limits are consistent with predictions from numerical simulations of CDM models, in which most of the gas is clumped in redshift space by this time (Croft et al. 1997; Zhang et al. 1997; Bi & Davidsen 1997). Although in principle the helium bounds are sound, the present results are given a "B" grade because they are as yet based on only two quasars.

4. NUCLEOSYNTHESIS

The observationally successful theory of the origin of the light elements by nucleosynthesis at redshift $z\sim 10^9$ predicts the mean baryon density in terms of the primeval element abundances (as reviewed extensively in the literature, e.g., Walker et al. 1991; Copi, Schramm, & Turner 1995; Hata et al. 1997). We consider here standard homogeneous nucleosynthesis predictions for abundances as a function of the baryon-to-photon ratio $\eta \equiv 10^{-10}\eta_{10}$, where the present baryon density is $\Omega_{\rm baryon}$ $h_{70}^2 = 7.45 \times 10^{-3}\eta_{10}$. In this context, the strongest constraints on $\Omega_{\rm baryon}$ derive from the abundances of helium and deuterium.

The primordial deuterium abundance is still uncertain, but we can quote reliable upper and lower bounds. A conservative lower bound, $(D/H)_n \ge 2 \times 10^{-5}$, comes from many sources—the Jovian atmosphere (e.g., Niemann 1996), the interstellar medium (Ferlet & Lemoine 1996; Linsky et al. 1995), and quasar absorption lines (Tytler, Fan, & Burles 1996), together with the fact that no source outside the Big Bang is known to produce deuterium significantly. An upper bound, $(D/H)_p \le 2 \times 10^{-4}$, comes from several measurements in metal-poor quasar absorption systems (Songaila et al. 1994; Carswell et al. 1994; Webb et al. 1997; Songaila, Wampler, & Cowie, 1997). Although the identification as deuterium in these systems is disputed (Tytler, Burles, & Kirkman 1997), the lack of higher detected values and the fact that significant D destruction would normally be expected to produce significant metal enrichment makes this a robust upper limit. These yield the limits $1.7 \le \eta_{10} \le 7.2$. The lower bound is used for the minimum value of Ω_{baryon} in line 14 of Table 3. For comparison, we include the central value favored by Burles & Tytler (1997), $(D/H)_p = 3.4 \times 10^{-5}$, which yields $\eta_{10} = 5.1$ or $\Omega_{\rm baryon} = 0.039 \, h_{70}^{-2}$.

The helium abundance Y is well measured in nearby galaxies (e.g., Pagel et al. 1992; Skillmann et al. 1994; Izotov, Thuan, & Lipovetsky 1997). The primordial value Y_p derived from these observations depends on models of stellar enrichment, but the present data sets yield a nearly model-independent 2 σ Bayesian upper limit of $Y_p \leq 0.243$ (Hogan, Olive, & Scully 1997). This corresponds to $\eta_{10} = 3.6$, which we use for the maximum value in line 13 of Table 3; it is about half the upper limit from deuterium. Most current studies (e.g., Olive & Steigman 1995) are consistent with the central value $Y_p \simeq 0.23$ used in this table.

5. DISCUSSION

After a brief comparison with earlier work (§ 5.1), we offer in § 5.2 an assessment of the major uncertainties in our budget at low redshift, with emphasis on the more indirect arguments that we hope will constrain the budget of baryons in forms that are unobservable in practice. We review the test from the theory of the origin of the light elements in § 5.3, and in § 5.4 we compare these results with measurements of the diffuse baryons at $z \approx 3$. Some concluding remarks are offered in § 5.5.

5.1. Comparison with Earlier Estimates

Let us compare our estimates with those of Persic & Salucci (1992), Gnedin & Ostriker (1992), and Bristow & Phillipps (1994). These three groups considered baryons in stars (often classified into those in early and late types of galaxies) and in hot X-ray-emitting gas around clusters. The baryon abundance in stars estimated by Gnedin & Ostriker is in good agreement with ours, and that of Persic & Salucci is close to our minimum estimate. On the other hand, the estimate of Bristow & Phillipps is 6 times ours. The same pattern applies to baryons in X-ray-emitting gas: Persic & Salucci obtained $\frac{1}{3}$ our central value and Bristow & Phillipps found twice our estimate. As a result, Persic & Salucci find that the total amount of baryons is substantially less than expected from nucleosynthesis, while Bristow & Phillipps find that the nucleosynthesis value is saturated by stars plus hot X-ray-emitting gas alone. Our attempt to take into account a broader range of constraints and states of baryons, including plasma in groups inferred but not directly observed, has led us to conclude that, apart from baryons that may have been sequestered at very high redshift, the bulk are either directly visible or in forms that are well constrained and accounted for with plausible extrapolations.

5.2. Observable and Unobservable Baryons

A baryon budget must be informed by an understanding of what is reasonable and sensible within standard models for the physics, astronomy, and cosmology. We argue here that these considerations coupled with the observations available now or within reach significantly limit the possibilities for large gaps in the budget.

Chemically Bound Baryons.—Baryons confined by chemical bonds, as in comets, are dark (apart from evaporation) at any significant distance, but we know that their contribution to the baryon mass within the luminous parts of normal galaxies is not large; the mass in stars accounts for the dynamics. In the standard cosmology, only a small fraction of primordial hydrogen converts to molecular form. Some scenarios envisage a significant mass in molecular clouds (Combes & Pfenniger 1998), but it seems more likely to us that the bulk of this material would by now have formed into stars, as it has in the Galaxy. Baryons could be chemically bound to heavier elements, but it is hard to see how this could be a significant mass component because it would require prodigious heavy element production in closely confined and closed systems, and the radiated nuclear binding energy would appear in the background radiation. In addition to these arguments, we have direct limits on the density of the plentiful but inert gas helium that would separate from any condensed component. In short, if the existence of a significant mass in intergalactic comets could be established, it would contradict the standard cosmological model. The far more credible proposition is that there is not a significant mass fraction in chemically bound baryons.

Dark Galaxies.—The starlight from baryons gravitationally bound in known forms of low surface brightness (LSB) galaxies is only about 15% of our adopted total. Thus, if normal galaxies were an adequate guide to their compositions, the known LSB galaxies would be insignificant reservoirs of baryons in stars. The discovery of a reason to suspect the contrary would be of great interest, of course. The starlight in as yet undetected still lower surface

brightness galaxies is constrained by the absolute surface brightness of the sky and fluctuations thereof; a significant contribution already appears unlikely from the smoothness of the background in deep exposures. If measurements indicated that the mean optical extragalactic surface brightness is larger than expected from known galaxies, it certainly would give reason to think that we have missed an important baryon component.

If the surface brightness showed no such anomaly, one could still imagine that there are dark galaxies with star populations dominated by brown dwarfs, but there are several arguments against this idea. There are high success rates in optical identification of systems of normal galaxies with the objects responsible for gravitational lensing events for quasar absorption line systems and for X-ray sources. Even if massive dark galaxies were assumed not to be capable of significant lensing, they could contain diffuse baryonic halos, so in this scenario we would need to explain why they do not cause a significant rate of identification failures for quasar absorption lines and X-ray sources. One way assumes that the dark galaxies avoid the concentrations of normal galaxies and the associated plasma, and so do not acquire plasma halos, but there are two counterarguments. First, in this picture there surely would be intermediate cases, dim massive galaxies that avoid the normal ones, and there is no evidence of them. Second, the familiar morphology-density relation in observed galaxies goes the other way: gas-rich galaxies prefer lower density environments, while gas-poor anemic spirals prefer denser regions.

Dust-Shrouded Stars.—Populations of stars shrouded in dust are constrained by the integrated far-infrared background of reradiated starlight, which is observed to have an integrated flux about twice that from optical galaxies as estimated from the Hubble Deep Field, and which is thus the repository of most of the nuclear energy released by stars (Schlegel, Finkbeiner, & Davis 1998; Puget et al. 1996; Fixen et al. 1998). The total mass in baryons in stars, even allowing for a maximal shrouded population, must, however, still be small. It is also interesting to compare the estimated total energy density with the corresponding global production of heavy elements. Taking the total bolometric intensity of starlight to be about 50 n Wm $^{-2}$ sr $^{-1}$ (based on Schlegel et al.'s fitted value of 32 ± 13 at $140 \mu m$), the energy density is

$$u = 2 \times 10^{-14} (1 + z) \text{ ergs cm}^{-3}$$

= 1 × 10⁻⁸(1 + z) MeV cm⁻³, (36)

where the redshift factor corrects for energy lost after the mean epoch of emission. Each nucleon that is burned to helium releases 25 MeV in heat, rising to a total of about 30 MeV per nucleon converted from hydrogen to heavy elements. On dividing u by 30 MeV, we get the mean nucleon number density in heavy elements produced in the production of the background light, $n_{\text{heavy}} \sim 4 \times 10^{-10} \, (1+z) \, \text{cm}^{-3}$. Our central value for the baryon number density is $n_{\text{baryons}} = 1.1 \times 10^{-7} \, \text{cm}^{-3}$; the ratio is the mass fraction in heavy elements to make the observed background,

$$Z \sim 0.004(1+z)$$
. (37)

If the bulk of the radiation were produced at z=2, it would mean that Z is about 1%. Although not a precise constraint, this is a significant check that the number roughly agrees with the observed metallicity of baryonic material.

Black Holes.—Baryons sequestered in black holes in the luminous parts of normal galaxies are known to be subdominant to the baryons in stars. It is quite unreasonable to imagine that whole galaxies have been lost to relativistic collapse. Gas clouds at the Jeans length at decoupling may be susceptible to relativistic collapse, but only a tiny fraction of material has small enough angular momentum to form black holes, and even then it is hard to imagine that an appreciable fraction of the cloud mass could collapse before the remainder is blown apart by radiation from the collapsing fraction. Far before recombination, even a tiny collapsing fraction yields a significant mass fraction today, although it requires either extremely large amplitude perturbations in η , which tends to adversely affect the model for light element production (e.g., Kurki-Suonio, Jedamzik, & Mathews 1996), or extreme fine tuning of the spectrum and amplitude of small-scale adiabatic perturbations (Carr 1994). In these scenarios, baryons may be sequestered in compact objects at redshifts $z \gtrsim 10^{10}$ that are in effect nonbaryonic for the purpose of this paper.

Baryonic MACHOs.—Debris is a key issue for MACHOs interpreted as star remnants. In known processes of formation of white dwarfs and neutron stars, winds and explosions disperse most of the original star mass in diffuse debris. Thus, a 2 M_{\odot} star leaves a 0.5 M_{\odot} remnant and 1.5 M_{\odot} in debris, and the ejected fraction is larger for larger stars. The lensing observations are consistent with a density parameter $\Omega \sim 0.25$ in MACHOs, which would be a very considerable entry in the budget, but we would have to explain what happened to the debris. The debris might be recycled and a large fraction ultimately sequestered in many generations of MACHOs, but recycling seems unlikely in the low-density halos of galaxies. Debris still present as diffuse matter, with a density parameter $\Omega \geq 0.014$, would violate constraints previously discussed on diffuse matter in groups, and most models would also generate excessive metals. The most plausible form of baryons is brown dwarfs, though this would require a second peak in the global IMF. Brown dwarf masses are below the estimated mass range of the observed MACHOs, but this could be the result of an unfortunate distribution of velocities of the MACHOs in the direction of the LMC. Absent the discovery of observational evidence for the brown dwarf picture or a demonstration that the formation of star remnants need not disperse much debris in observable forms, we suspect that the MACHO contribution to the baryon budget is subdominant to the mass in plasma around

Plasma around Field Galaxies.—There is a significant baryon density in stars in normal galaxies, and a still larger density in plasma around the galaxies. The latter is the largest and most uncertain entry in our baryon budget at low redshift in Table 3. The estimate in line 7' assumes that the spheroid star-to-gravitational mass ratio is the same in clusters and the field. If we have missed a significant mass in spheroids, as in LSB galaxies, and the missing fraction is the same in clusters and the field, it does not affect the estimate. If spheroid star production were more efficient in clusters, it would mean that the estimated group mass is too small and hence line 7' is too small. If plasma were ejected (more than spheroid stars) from clusters during assembly, line 7' would be an overestimate. But neither of these effects could be large without upsetting the concordance between equations (24), (25), and (27). Comparison of the two estimates, line 7',

and the sum of lines 7a and 7b shows a factor of 2 difference, but it is easy to imagine that this is because the X-ray observations still miss considerable warm plasma, and we have not yet adequately modeled the cooler low surface density clouds detected by absorption lines at low redshift.

Baryons in the Voids.—There are galaxies in the low-density regions, or voids, defined by normal galaxies. Galaxies in low-density regions tend to be later Hubble types, and so are more readily observable than their counterparts in concentrations. Gas enrichment aside, no known type of object prefers the voids. This includes dwarf and irregular galaxies, low surface brightness galaxies, and the plasma clouds detected from absorption lines. The reasonable presumption is that plasma clusters in the same way that all other observed baryons do, meaning that there is not much mass in plasma in the voids. The possible exception is gas with high enough pressure to resist gravitational draining of the voids, but if the density of such void plasma were comparable to that estimated in groups, it would have appeared already as excessive helium absorption at $z \approx 3$.

The lack of reasonable-looking alternatives leads us to conclude that a fair accounting of the baryons is possible, because most are in states that can be observed or reliably constrained by more indirect arguments. We now argue that there is a reasonable case for the net baryon density parameter

$$\Omega_{\rm baryon} = 0.021 \pm 0.007 ,$$
(38)

close to the central value of the sum in line 8 of Table 3.

5.3. The Nucleosynthesis Check

The central density in equation (38) with $h_{70} = 1$ corresponds to $\eta_{10} = 2.8$. At this value of the baryon-to-photon ratio, the standard model for light element nucleosynthesis predicts $Y_p(^4\text{He}) \simeq 0.238$ and D/H $\simeq 9 \times 10^{-5}$. This is consistent with observations of the helium abundance, if somewhat above the central value, and with current observations of the deuterium abundance, if somewhat toward the high end of the range of estimates (Songaila 1997). The mean of the upper limit from helium and the lower limit from deuterium is $\eta_{10} = 2.7$, which we adopt as the central value in line 14 of Table 3.

If the baryon density were at the low end of our range of estimates in line 8 of Table 3, $\Omega_{\rm baryon}=0.007$ ($\eta_{10}=1.0$ at $h_{70}=1$), the standard model prediction for the deuterium abundance would be unacceptably large. This could be remedied by assuming that a significant baryon mass has been sequestered in the MACHOs, but for the reasons explained above we are inclined to suspect rather that the adopted minima in lines 7a and 7b of Table 3 seriously underestimate the mass in plasma. The baryon density at the high end of our estimates is $\Omega_{\rm baryon}=0.041$ ($\eta_{10}=5.5$ at $h_{70}=1$). This density is consistent with the bound from the deuterium abundance, but it predicts $Y_p=0.247$, well outside the accepted bound on the primeval helium mass fraction. In our opinion, the likely interpretation is that our upper limit in line 8 of Table 3 is also overly conservative.

Our interpretation of the baryon budget assumes the standard homogeneous model for primordial nucleosynthesis. An inhomogeneous primeval entropy per baryon, with the appropriate coherence length (Jedamzik & Fuller 1995), would require serious attention if further observations confirmed that there are significant variations in the

deuterium abundances in high-redshift H I clouds or that the helium abundances in some dwarf galaxies are lower than that predicted by the global baryon budget applied to the homogeneous nucleosynthesis model.

5.4. The Check from the Budget at $z \approx 3$

Stars are subdominant in the budget at low redshift and are likely even less significant at redshift $z \approx 3$, so it makes sense to compare the sum of the high-redshift diffuse gas components in Table 3 to the sum at low redshift and the prediction of light element nucleosynthesis.

The sum of the central values for diffuse baryons at $z \approx 3$ is $\Omega = 0.04$. This is consistent with the maximum sum in our low-redshift budget, but again contradicts the standard model for helium abundance. The sum of the minimum values at $z \approx 3$ is within the error flags in equation (38) and the standard model for the light elements. Thus, there is no contradiction with equation (38), but advances in observations of the intergalactic medium at high redshift may provide a critical test.

5.5. Concluding Remarks

We have emphasized three themes. First, a robust estimate of the present baryon budget is of central importance in serving to clarify and sharpen issues of research in present-day cosmology. Second, there is no guarantee that physical processes in the formation and evolution of structure will have conspired to place a significant fraction of the baryons in forms we can hope to detect by their emission or absorption of radiation or otherwise infer in a convincing way. Third, the developing evidence is that Nature in fact has been kind: we may actually be able to arrive at a close to complete budget of the baryons in the observable universe.

The value for Ω_{baryon} in equation (38) suggests or is consistent with three lines of ideas. First, it suggests that baryonic MACHOs make an insignificant contribution to the global budget. Perhaps the MACHOs are nonbaryonic entities formed in the early universe, or perhaps like the stellar components of the Milky Way galaxy they are more concentrated in the inner halo than is nonbaryonic dark matter. The second idea is that there are relatively few baryons in the voids defined by the galaxies, likely because the baryons were swept out of the voids by gravity. Gravity would gather low-pressure dark matter with the baryons and galaxies, implying that the mass density in matter that can cluster is well below the Einstein-de Sitter value. This is consistent with our estimate of the total density parameter in equation (24). The third idea is that the bulk of the forest baryons at z = 3 have ended up now in warm plasma around the galaxies outside the great clusters. The mass in neutral hydrogen in the DLAs is about half that needed to account for the present-day mass in stars. We know that some mass is already in stars at z = 3; formation of the rest of the star mass at z < 3 would be a modest drain on the mass in the forest.

An interesting feature of the budget is that all stars and their remnants, together with cold phases of collapsed gas, comprise a small fraction even of the small fraction of the matter that is baryonic (Gnedin & Ostriker 1992; Persic & Salucci 1992). Just the 0.3% of the baryons in irregular galaxies shine brightly enough at z=0.4-1 to make an important contribution to the rapid increase of B-band

number counts of galaxies with increasing apparent magnitude. It would not be difficult in these circumstances to imagine that the distribution of the starlight has little to do with the mass. But the consistency of the estimates of dynamical estimates of the total density parameter, as in equations (24), (25), and (27), suggests that starlight nevertheless is a useful tracer of mass on the scale of the distance between galaxies.

The baryon budget still is quite uncertain; the sums in line 8 of Table 3 for the maximum and minimum low-redshift budget differ by a factor of 6. We suspect that this is largely a result of an underestimate in line 7a for plasma in groups; improved information on soft X-ray emission would be of considerable help. Perhaps improvements in the observational constraints will confirm the tentative con-

cordance in equation (38); perhaps better observations will reveal an inconsistency that shows we have to reconsider some aspect of the standard concepts of astronomy and cosmology.

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